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KAON AND CHARM PHYSICS: THEORY

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We introduce and discuss basic topics in the theory of kaons and charmed particles. In the first part, theoretical methods in weak decays such as operator product expansion, renormalization group and the construction of effective Hamiltonians are presented, along with an elementary account of chiral perturbation theory. The second part describes the phenomenology of the neutral kaon system, CP violation, ε and ε'/ε , rare kaon decays ($K \rightarrow \pi\nu\bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$, $K_L \rightarrow \mu^+ \mu^-$), and some examples of flavour physics in the charm sector.

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1 Preface

These lectures provide an introduction to the theory of weak decays of kaons and mesons with charm. Our main focus will be on kaon physics, which has led to many deep and far-reaching insights into the structure of matter, is a very active field of current research and still continues to hold exciting opportunities for future discoveries. Another, and in several ways complementary source of information about flavour physics is the charm sector. Standard model effects for rare processes are in this case typically suppressed to almost negligible levels and positive signals, if observed, could therefore yield spectacular evidence of new physics. Towards the end of the lectures we will describe a few selected examples in charm physics and contrast their characteristic features with those of the kaon sector. For both subjects we will concentrate on the flavour physics of the standard model. We discuss the phenomenology as well as the theoretical tools necessary to achieve a detailed and comprehensive test of the standard model picture that should eventually lead us to uncover signals of the physics beyond. The experimental aspects of these fields are explained in the lectures by Barker (kaon physics) and Cumalat (charm physics) at this School. Before we start our tour of flavour physics with kaons and charm, we give a brief outline of the contents of these lectures.

We begin, in the following section 2, with recalling some of the historical highlights of kaon physics and with an overview of the main topics of current interest in this field.

In section 3 we introduce theoretical methods that are fundamental for the computation of weak decay processes and for relating the basic parameters of the underlying theory to actual observables. These important tools are the operator product expansion, the renormalization group, the effective low-energy

weak Hamiltonians, where we discuss the general $\Delta S = 1$ Hamiltonian as an explicit example, and, finally, chiral perturbation theory. Our main emphasis will be on an elementary introduction of the relevant ideas and concepts, rather than on more specialized technical aspects.

With this background in mind we will then address, in section 4, the phenomenology of the neutral-kaon system and CP violation. We discuss a common classification of CP violation, the kaon CP parameters ε and ε'/ε , and the standard analysis of the CKM unitarity triangle.

Section 5 is devoted to the physics of rare kaon decays, in particular the “golden” channels $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$, and the processes $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \mu^+ \mu^-$.

In section 6 we discuss the prominent features of flavour physics with charm. We present some opportunities with rare decays of D mesons and describe the phenomenology of D^0 – \bar{D}^0 mixing, which is of current interest in view of new experimental measurements by the CLEO and FOCUS collaborations.

Finally, section 7 summarizes the main points and presents an outlook on future opportunities.

We conclude these preliminary remarks by mentioning several review articles, which the interested reader may consult for further details on the topics presented here, for a discussion of related additional processes and for a complete collection of references to the original literature. Very useful accounts of rare and radiative kaon decays and of kaon CP violation can be found in ^{1,2,3,4,5,6,7}. The first five articles also discuss the relevant experiments. Nice reviews on flavour physics with charm are ^{8,9,10,11}. Further details on theoretical methods in weak decays are provided in ^{12,13}.

2 Kaons: Introduction and Overview

2.1 Historical Highlights

The history of kaon physics is remarkably rich in groundbreaking discoveries. It will be interesting to briefly recall some of the most exciting examples here. We do not attempt to give an historically accurate account of the development of kaon physics. This is a fascinating subject in itself. For a more complete historical picture the reader may consult the excellent book by Cahn and Goldhaber¹⁴. Here we will content ourselves with a brief sketch of several highlights related to kaon physics. They serve to illustrate how the observation of unexpected – and sometimes tiny – effects in this field is linked to basic concepts in our theoretical understanding of the fundamental interactions.

Strangeness

Already the discovery of kaons alone, half a century ago, has had an impressive impact on the development of high-energy physics. One of the characteristic features of the new particles was *associated production*, that is they were always produced in pairs by strong interactions, for instance as

$$\begin{array}{cccccc} \pi^+ & + & p & \rightarrow & K^+ & + & \bar{K}^0 & + & p \\ \text{S} & & 0 & & 0 & & +1 & & -1 & & 0 \end{array}$$

(Alternatively a K^+ could be produced along with a $\Lambda(uds)$ baryon.) This property, together with the long lifetime, suggested the existence of a new quantum number, called strangeness, carried by the kaons and conserved in strong interactions. The discovery of strangeness opened the way for the $SU(3)$ classification of hadrons and the introduction of quarks (u , d , s) as the fundamental representation by Gell-Mann. The quark picture, in turn, formed the basis for the subsequent development of QCD. In modern notation the K mesons come in the following varieties

$$\begin{array}{cc} K^+(\bar{s}u) & K^0(\bar{s}d) \\ K^-(s\bar{u}) & \bar{K}^0(sd) \end{array}$$

where the flavour content is indicated in brackets. The pairs (K^+, K^0) and (\bar{K}^0, K^-) are doublets of isospin.

Parity Violation

The new mesons proved to be strange particles indeed. One of the peculiarities is known as the θ - τ puzzle. Two particles decaying as $\theta \rightarrow 2\pi$ (P even final state) and $\tau \rightarrow 3\pi$ (P odd), and hence apparently of different parity, were observed to have the same mass and lifetime. This situation prompted Lee and Yang to propose that parity might not be conserved in weak interactions. This was later confirmed in the famous ^{60}Co experiment by C.S. Wu. Today the θ^+ and τ^+ are known to be identical to the K^+ meson and parity violation is firmly encoded in the chiral $SU(2)_L$ gauge group of standard model weak interactions.

CP Violation

After the recognition of parity violation in weak processes the combination of parity with charge conjugation, CP, still appeared to be a good symmetry. The neutral kaons K^0 and \bar{K}^0 were known to mix through second order weak interactions to form, if CP was conserved, the CP eigenstates $K_{L,S} = (K^0 \pm \bar{K}^0)/\sqrt{2}$ (here $CP K^0 \equiv -\bar{K}^0$). Clearly, CP symmetry then forbids the decay

of the CP-odd K_L into the CP-even $\pi^+\pi^-$ final state. Instead, Christenson, Cronin, Fitch and Turlay showed in 1964 that the decay does in fact occur, establishing CP violation. Compared to the CP-allowed decay of $K_S \rightarrow \pi^+\pi^-$ the amplitude is measured to be

$$\left| \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} \right| = 2.3 \cdot 10^{-3} \quad (1)$$

CP violation is thus a very small effect, in contrast to P violation, but the *qualitative* implications are nevertheless far-reaching. As we will discuss later, CP violation defines an absolute, and not only conventional, difference between matter and anti-matter. Also, as we now know, CP violation indirectly anticipated in a sense the need for three families of fermions within the standard model. Finally, CP violation is a necessary prerequisite for the generation of a net baryon number in our universe according to Sakharov's three conditions (the other two being baryon number violation and a departure from thermal equilibrium).

FCNC Suppression

Another striking property of weak interactions that manifested itself in kaon decays is the suppression of flavour-changing neutral currents (FCNC). While the standard, charged-current mediated process $K^+ \rightarrow \mu^+\nu$ has a branching fraction of order unity

$$B(K^+ \rightarrow \mu^+\nu) = 0.64 \quad (2)$$

the similarly looking neutral-current decay $K_L \rightarrow \mu^+\mu^-$ is suppressed to a tiny level

$$B(K_L \rightarrow \mu^+\mu^-) \approx 7 \cdot 10^{-9} \quad (3)$$

Naively, a “three-quark standard model” would allow a $\bar{s}dZ$ coupling at tree level. This would lead to a $K_L \rightarrow \mu^+\mu^-$ amplitude of strength G_F , comparable to $K^+ \rightarrow \mu^+\nu$, in plain disagreement with (3). Even if the tree-level coupling of $\bar{s}dZ$ were forbidden, the problem would reappear at one loop. This is illustrated in the second diagram of Fig. 1. The loop integral is divergent, where a natural cut-off could be expected at the weak scale $\sim M_W$. The amplitude should then be of the order $G_F^2 M_W^2$, which would still be far too large. Of course, the three-quark model is not renormalizable and therefore not a consistent theory at short distances. The introduction of the charm quark by Glashow, Iliopoulos and Maiani (GIM) solves all of these problems, which plagued the early theory of weak interactions. The complete two-generation standard model is perfectly consistent and the tree-level $\bar{s}dZ$ coupling is automatically eliminated by the orthogonality of the 2×2 Cabibbo mixing matrix.

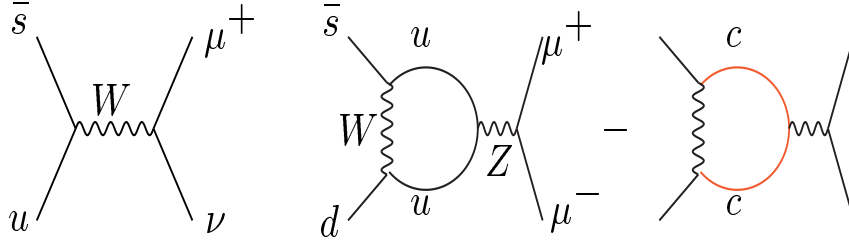


Figure 1: GIM Mechanism.

The $\bar{s}dZ$ coupling can still be induced at one-loop order, but the disturbing $G_F^2 M_W^2$ term is now canceled between the up-quark and the charm-quark contribution. The remaining effect is, up to logarithms, merely of the order $G_F^2 m_c^2$, which is well compatible with (3), unless m_c would be too large. To turn this observation into a more quantitative constraint on the charm-quark mass m_c is, however, not easy in this case because $K_L \rightarrow \mu^+ \mu^-$ is actually dominated by long-distance contributions (we will discuss this further in section 5.3). Another FCNC process, $K^0 - \bar{K}^0$ mixing, proved to be more useful in this respect.

$K - \bar{K}$ mixing, GIM and Charm

The following example represents one of the great triumphs of early standard model phenomenology. In the four-quark theory, $K - \bar{K}$ mixing occurs through $\Delta S = 2$ W -box diagrams with internal up and charm quarks. This $\Delta S = 2$ transition induces a tiny off-diagonal element M_{12} in the mass matrix

$$H_M = \begin{pmatrix} M & M_{12} \\ M_{12} & M \end{pmatrix} \quad (4)$$

of the $K - \bar{K}$ system. The corresponding eigenstates are $K_{L,S} = (K^0 \pm \bar{K}^0)/\sqrt{2}$ with eigenvalues $M_{L,S}$. The difference between the eigenvalues $\Delta M_K = M_L - M_S$ is related to M_{12} and can be estimated from the box diagrams (see Fig. 2). Anticipating a more detailed discussion of the calculation, we simply quote the result:

$$\frac{\Delta M_K}{M_K} \approx \frac{G_F^2 f_K^2}{6\pi^2} |V_{cs} V_{cd}|^2 m_c^2 = 7 \cdot 10^{-15} \quad (5)$$

where the number on the right is the experimental value. The theoretical expression is approximate since we have taken the required hadronic matrix

$$K_{L,S} = \frac{K^0 \pm \bar{K}^0}{\sqrt{2}} \quad \begin{array}{c} d \quad u, c \quad s \\ \text{---} \quad \text{---} \quad \text{---} \\ | \quad | \quad | \\ \text{W} \quad \text{W} \\ | \quad | \quad | \\ s \quad u, c \quad d \end{array} \quad \begin{pmatrix} M & M_{12} \\ M_{12} & M \end{pmatrix} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$$

Figure 2: K^0 – \bar{K}^0 mixing.

element in the so-called factorization approximation, we have neglected the (fairly small) up-quark contribution and assumed that $m_c \ll M_W$.

Qualitatively, (5) is easy to understand. There is a factor of G_F^2 representing the second order weak interaction and a factor of f_K^2 describing – roughly speaking – the bound state dynamics binding the s and d quarks into kaons. The factor $6\pi^2$ in the denominator is a typical numerical factor from the loop integration. The whole amplitude is proportional to m_c^2 , reflecting the remainder of a GIM cancellation (assuming $m_c \gg \Lambda_{QCD}$, the up-quark contribution is negligible). Finally there are the obvious CKM parameters.

This analysis was performed in 1974 by Gaillard and Lee. The charm quark had just been introduced for the theoretical reasons mentioned above, but had not yet been discovered in experiment. Gaillard and Lee realized that the quadratic dependence on the unknown charm quark mass in (5), resulting from the GIM cancellation of the $\Delta S = 2$ FCNC amplitude, could be turned into an estimate of this mass. They concluded that m_c should be about a few GeV. If you take the formula in (5) and put in numbers, you will find that $m_c \approx 1.5$ GeV (!). In view of the uncertainties of (5) the accuracy of this result cannot be taken too seriously, but it is in any case amusing that indeed a quite realistic charm mass comes out already from this simplified formula. What is more important, however, is the spirit of the argument, which led Gaillard and Lee to a correct prediction of m_c by taking the short-distance structure of the theory seriously. This was certainly one of the most beautiful successes of flavour physics with kaons.

All these examples illustrate that a careful analysis of low-energy processes such as kaon decays, can lead to truly profound insights into fundamental physics. Especially remarkable is the circumstance that kaon physics, which “operates” at the ~ 500 MeV scale, carries important information on the dynamics at much higher energy scales. The quark-structure of hadronic matter, the chiral nature of weak gauge interactions, the properties of charm, and

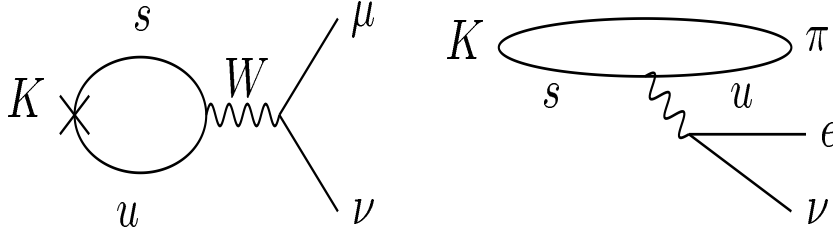


Figure 3: (Semi)leptonic K decays.

those of the top quark entering CP violating amplitudes, are prominent examples that illustrate this point. In fact, we see that several of the most crucial pillars of the standard model rest on results derived from studies with kaons. Of course, direct experiments at high energies, that have led for instance to the production of on-shell W and Z bosons or quark jets, are indispensable for exploring the structure of matter. However, indirect, low-energy precision observables are equally necessary as a complementary approach. They can yield information that is hardly accessible in any other way, such as the elucidation of the GIM structure of flavour physics or the violation of CP symmetry. It is with this philosophy in mind that studies of rare kaon processes, but also rare decays of b hadrons or charmed particles, continue to be pursued with great interest. The most promising future opportunities with kaons will be the subject of later sections in these lectures. We conclude this introductory chapter with a brief general overview of physics with kaon decays.

2.2 Overview of K decays

We may classify the decays of K mesons into several broad categories, some of which are more determined by nonperturbative strong interaction dynamics, while others have a high sensitivity to short-distance physics both in the standard model and beyond.

Tree-Level (Semi-) Leptonic Decays

These are the simplest decays of kaons. They typically have large branching ratios and are well studied. Examples are the purely leptonic decay $K^+ \rightarrow \mu^+ \nu$ and the semileptonic mode $K^+ \rightarrow \pi^0 e^+ \nu$, which are illustrated in Fig. 3. $K^+ \rightarrow \pi^0 e^+ \nu$ is very important for determining the CKM matrix element V_{us} (the sine of the Cabibbo angle). This is possible because the hadronic matrix

element of the vector current $\langle \pi^0 | (\bar{s}u)_V | K^+ \rangle$ is absolutely normalized in the limit of SU(3) flavour symmetry and protected from first order corrections in the SU(3) breaking (Ademollo-Gatto theorem). One finds

$$|V_{us}| = 0.2196 \pm 0.0023 \quad (6)$$

This is a basic input for the CKM matrix. Furthermore, knowing $|V_{us}|$, $K^+ \rightarrow \mu^+ \nu$ may be used to determine the kaon decay constant $f_K = 160 \text{ MeV}$.

Nonleptonic Decays

Nonleptonic decays, such as $K \rightarrow \pi\pi$, are strongly affected by nonperturbative QCD dynamics. Nevertheless they provide an important window on the violation of discrete symmetries, P and CP for example. CP violation is currently of special interest. It enters through $K-\bar{K}$ mixing via box graphs or through penguin diagrams in the decay amplitudes, with the virtual top quarks playing a decisive role. This is sketched in Fig. 4.

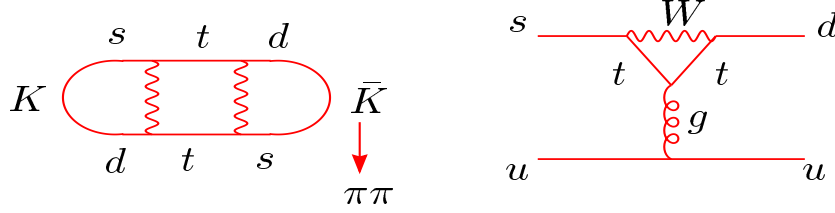


Figure 4: SM origin of CP violation in $K \rightarrow \pi\pi$ decays.

Long-Distance Dominated Rare and Radiative Decays

Examples of this class of processes are $K^+ \rightarrow \pi^+ l^+ l^-$, $K_L \rightarrow \pi^0 \gamma \gamma$, $K_S \rightarrow \gamma \gamma$ or $K_L \rightarrow \mu^+ \mu^-$. A typical contribution to $K^+ \rightarrow \pi^+ e^+ e^-$ is illustrated in Fig. 5. These processes are determined by nonperturbative low-energy strong interactions and can be analyzed in the framework of chiral perturbation theory. The treatment of nonperturbative dynamics within a first-principles approach as provided by chiral perturbation theory is of great interest in its own right. In addition, the control over long-distance contributions afforded by chiral perturbation theory can be helpful to extract information on the flavour physics from short distances.

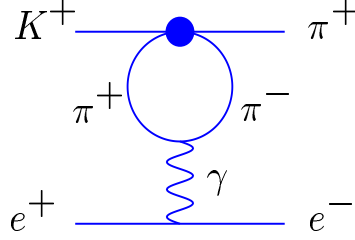


Figure 5: Typical contribution to $K^+ \rightarrow \pi^+ e^+ e^-$.

Short-Distance Dominated Rare Decays

The prime examples in this category are the processes $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Here short-distance dynamics completely dominates the decay and the access to flavour physics is very clean and direct. For this reason the $K \rightarrow \pi \nu \bar{\nu}$ modes are special highlights among the future opportunities in kaon physics. To a somewhat lesser extent also $K_L \rightarrow \pi^0 e^+ e^-$ qualifies for this class. In this case the fact that the process is predominantly CP violating enhances the sensitivity to short-distance physics in comparison to $K^+ \rightarrow \pi^+ e^+ e^-$.

Decays Forbidden in the SM

Any positive signal in processes that are forbidden in the standard model would be a very dramatic indication of new physics. A good example are kaon decays with lepton-flavour violation. Stringent experimental limits exist for several modes of interest

$$B(K_L \rightarrow \mu e) < 4.7 \cdot 10^{-12} \quad (7)$$

$$B(K^+ \rightarrow \pi^+ \mu^+ e^-) < 4.8 \cdot 10^{-11} \quad (8)$$

$$B(K_L \rightarrow \pi^0 \mu e) < 3.2 \cdot 10^{-9} \quad (9)$$

In principle these processes could be induced through loops in the standard model with neutrino masses. However, the smallness of the neutrino masses compared to the weak scale results in unmeasurably small branching fractions of typically below 10^{-25} . Larger values can be obtained within the minimal supersymmetric standard model (MSSM). However, there are strong constraints from direct limits on $\mu \rightarrow e$ conversion processes ($\mu \rightarrow e \gamma$ decay, or $\mu \rightarrow e$ conversion in the field of a nucleus). The disadvantage of $K_L \rightarrow \mu e$ is that flavour violation is needed simultaneously both in the lepton sector and in the

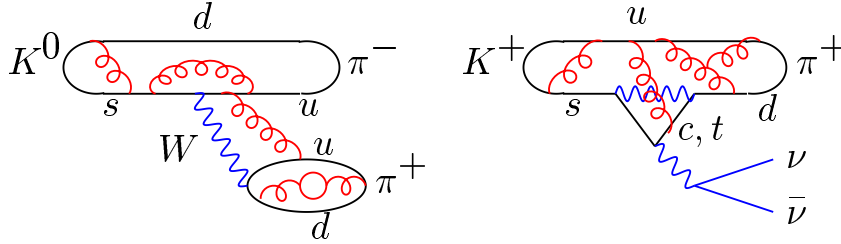


Figure 6: QCD effects in weak decays.

quark sector. Interesting effects could however still occur in some regions of parameter space. Systematically larger branching ratios are allowed in scenarios with R-parity violation, where decays such as $K_L \rightarrow \mu e$ can proceed at tree level¹⁵.

In very general terms, a scenario where the exchange of a heavy boson X mediates $\bar{s}d \rightarrow \bar{\mu}e$ transitions at tree level receives strong constraints from the tight experimental bound (7). Assuming couplings of electroweak strength, the bound (7) implies a lower limit of the X mass $M_X \gtrsim 100 \text{ TeV}$. Such a sensitivity to high energy scales is very impressive, however one has to remember that the tree-level scenario assumed above is quite simple-minded and in general very model dependent. Generically, one would expect some additional suppression mechanism to be at work in $K_L \rightarrow \mu e$. In this case the scale probed would be less, but the high precision of (7) still guarantees an excellent sensitivity to subtle short-distance effects.

3 Theoretical Methods in Weak Decays

The task of computing weak decays of kaons represents a complicated problem in quantum field theory. Two typical cases, the first-order nonleptonic process $K^0 \rightarrow \pi^+ \pi^-$, and the loop-induced, second-order weak transition $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ are illustrated in Fig. 6. The dynamics of the decays is determined by a nontrivial interplay of strong and electroweak forces, which is characterized by several energy scales of very different magnitude, the W mass, the various quark masses and the QCD scale: $m_t, M_W \gg m_c \gg \Lambda_{QCD} \gg m_u, m_d, (m_s)$. While it is usually sufficient to treat electroweak interactions to lowest nonvanishing order in perturbation theory, it is necessary to consider all orders in QCD. Asymptotic freedom still allows us to compute the effect of strong interactions at short distances perturbatively. However, since kaons are bound

states of light quarks, confined inside the hadron by long-distance dynamics, it is clear that also nonperturbative QCD interactions enter the decay process in an essential way.

To deal with this situation, we need a method to disentangle long- and short-distance contributions to the decay amplitude in a systematic fashion. The required tool is provided by the operator product expansion (OPE).

3.1 Operator Product Expansion

We will now discuss the basic concepts of the OPE for kaon decay amplitudes. These concepts are of crucial importance for the theory of weak decay processes, not only of kaons, but also of mesons with charm and beauty and other hadrons as well. Consider, for instance, the basic W -boson exchange process shown on the left-hand side of Fig. 7. This diagram mediates the

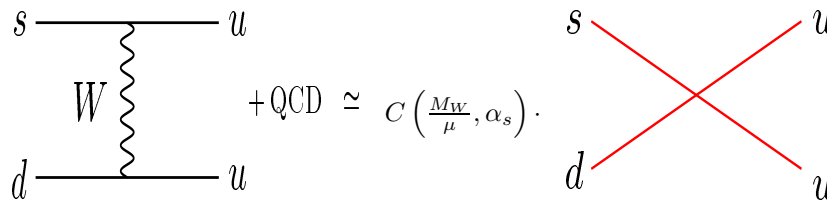


Figure 7: OPE for weak decays.

decay of a strange quark and triggers the nonleptonic decay of a kaon such as $K^0 \rightarrow \pi^+ \pi^-$. The quark-level transition shown is understood to be dressed with QCD interactions of all kinds, including the binding of the quarks into the mesons. To simplify this problem, we may look for a suitable expansion parameter, as we are used to do in theoretical physics. Here, the key feature is provided by the fact that the W mass M_W is very much heavier than the other momentum scales p in the problem (Λ_{QCD} , m_u , m_d , m_s). We can therefore expand the full amplitude A , schematically, as follows

$$A = C \left(\frac{M_W}{\mu}, \alpha_s \right) \cdot \langle Q \rangle + \mathcal{O} \left(\frac{p^2}{M_W^2} \right) \quad (10)$$

which is sketched in Fig. 7. Up to negligible power corrections of $\mathcal{O}(p^2/M_W^2)$, the full amplitude on the left-hand side is written as the matrix element of a local four-quark operator Q , multiplied by a Wilson coefficient C . This expansion in $1/M_W$ is called a (short-distance) operator product expansion because

the nonlocal product of two bilinear quark-current operators $(\bar{s}u)$ and $(\bar{u}d)$ that interact via W exchange, is expanded into a series of local operators. Physically, the expansion in Fig. 7 means that the exchange of the very heavy W boson can be approximated by a point-like four-quark interaction. With this picture the formal terminology of the OPE can be expressed in a more intuitive language by interpreting the local four-quark operator as a four-quark interaction vertex and the Wilson coefficient as the corresponding coupling constant. Together they define an effective Hamiltonian $\mathcal{H}_{eff} = C \cdot Q$, describing weak interactions of light quarks at low energies. Ignoring QCD the OPE reads explicitly (in momentum space)

$$\begin{aligned} A &= \frac{g_W^2}{8} V_{us}^* V_{ud} \frac{i}{k^2 - M_W^2} (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} \\ &= -i \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} C \cdot \langle Q \rangle + \mathcal{O}\left(\frac{k^2}{M_W^2}\right) \end{aligned} \quad (11)$$

with $C = 1$, $Q = (\bar{s}u)_{V-A} (\bar{u}d)_{V-A}$ and

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} \quad (12)$$

As we will demonstrate in more detail below after including QCD effects, the most important property of the OPE in (10) is the *factorization* of long- and short-distance contributions: All effects of QCD interactions above some factorization scale μ (short distances) are contained in the Wilson coefficient C . All the low-energy contributions below μ (long distances) are collected into the matrix elements of local operators $\langle Q \rangle$. In this way the short-distance part of the amplitude can be systematically extracted and calculated in perturbation theory. The problem to evaluate the matrix elements of local operators between hadron states remains. This task requires in general nonperturbative techniques, as for example lattice QCD, but it is considerably simpler than the original problem of the full standard-model amplitude. In some cases also symmetry considerations can help to determine the nonperturbative input. For example, the only matrix element relevant for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is

$$\langle \pi^+ | (\bar{s}d)_V | K^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}u)_V | K^+ \rangle \quad (13)$$

where the equality with the right-hand side uses isospin symmetry and allows us to obtain the matrix element from measuring the standard semileptonic mode $K^+ \rightarrow \pi^0 l^+ \nu$.

The short-distance OPE that we have described, the resulting effective Hamiltonian, and the factorization property are fundamental for the theory of

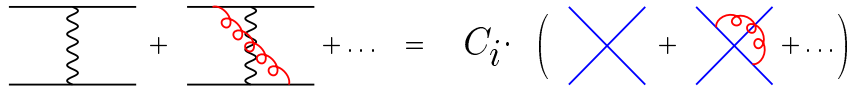


Figure 8: Calculation of Wilson coefficients of the OPE.

K decays. However, the concept of factorization of long- and short-distance contributions reaches far beyond these applications. In fact, the idea of factorization, in various forms and generalizations, is the key to essentially all applications of perturbative QCD, including the important areas of deep-inelastic scattering and jet or lepton pair production in hadron-hadron collisions. The reason is the same in all cases: Perturbative QCD is a theory of quarks and gluons, but those never appear in isolation and are always bound inside hadrons. Nonperturbative dynamics is therefore always relevant to some extent in hadronic reactions, even if these occur at very high energy or with a large intrinsic mass scale (see also the lectures by Soper in these proceedings). Thus, before perturbation theory can be applied, nonperturbative input has to be isolated in a systematic way, and this is achieved by establishing the property of factorization. It turns out that the weak effective Hamiltonian for K decays provides a nice example to demonstrate the general idea of factorization in simple and explicit terms.

We will next discuss the OPE for K decays, now including the effects of QCD, and illustrate the calculation of the Wilson coefficients. A diagrammatic representation for the OPE is shown in Fig. 8. The key to calculating the coefficients C_i is again the property of factorization. Since factorization implies the separation of all long-distance sensitive features of the amplitude into the matrix elements of $\langle Q_i \rangle$, the short-distance quantities C_i are, in particular, independent of the external states. This means that the C_i are always the same, no matter whether we consider the actual physical amplitude where the quarks are bound inside mesons, or any other, unphysical amplitude with on-shell or even off-shell external quark lines. Thus, even though we are ultimately interested in $K \rightarrow \pi\pi$ amplitudes, for the perturbative evaluation of C_i we are free to choose any treatment of the external quarks according to our calculational convenience. A convenient choice that we will use below is to take all light quarks massless and with the same off-shell momentum p ($p^2 \neq 0$).

The computation of the C_i in perturbation theory then proceeds in the following steps:

- Compute the amplitude A in the full theory (with W propagator) for

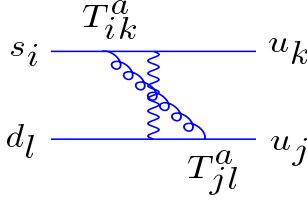


Figure 9: QCD correction with colour assignment.

arbitrary external states.

- Compute the matrix elements $\langle Q_i \rangle$ with the same treatment of external states.
- Extract the C_i from $A = C_i \langle Q_i \rangle$.

We remark that with the off-shell momenta p for the quark lines the amplitude is even gauge dependent and clearly unphysical. However, this dependence is identical for A and $\langle Q_i \rangle$ and drops out in the coefficients. The actual calculation is most easily performed in Feynman gauge. To $\mathcal{O}(\alpha_s)$ there are four relevant diagrams, the one shown in Fig. 8 together with the remaining three possibilities to connect the two quark lines with a gluon. Gluon corrections to either of these quark currents need not be considered, they are the same on both sides of the OPE and drop out in the C_i . The operators that appear on the right-hand side follow from the actual calculations. Without QCD corrections there is only one operator of dimension 6

$$Q_2 = (\bar{s}_i u_i)_{V-A} (\bar{u}_j d_j)_{V-A} \quad (14)$$

where the colour indices have been made explicit. (The operator is termed Q_2 for historical reasons.) To $\mathcal{O}(\alpha_s)$ QCD generates another operator

$$Q_1 = (\bar{s}_i u_j)_{V-A} (\bar{u}_j d_i)_{V-A} \quad (15)$$

which has the same Dirac and flavour structure, but a different colour form. Its origin is illustrated in Fig. 9, where we recall the useful identity for $SU(N)$ Gell-Mann matrices

$$(\bar{s}_i T_{ik}^a u_k)(\bar{u}_j T_{jl}^a d_l) = -\frac{1}{2N}(\bar{s}_i u_i)(\bar{u}_j d_j) + \frac{1}{2}(\bar{s}_i u_j)(\bar{u}_j d_i) \quad (16)$$

It is convenient to employ a different operator basis, defining

$$Q_{\pm} = \frac{Q_2 \pm Q_1}{2} \quad (17)$$

The corresponding coefficients are then given by

$$C_{\pm} = C_2 \pm C_1 \quad (18)$$

If we denote by S_{\pm} the spinor expressions that correspond to the operators Q_{\pm} (in other words: the tree-level matrix elements of Q_{\pm}), the full amplitude can be written as

$$A = \left(1 + \gamma_+ \alpha_s \ln \frac{M_W^2}{-p^2}\right) S_+ + \left(1 + \gamma_- \alpha_s \ln \frac{M_W^2}{-p^2}\right) S_- \quad (19)$$

Here we have focused on the logarithmic terms and dropped a constant contribution (of order α_s , but nonlogarithmic). Further, p^2 is the virtuality of the quarks and γ_{\pm} are numbers that we will specify later on. We next compute the matrix elements of the operators in the effective theory, using the same approximations, and find

$$\langle Q_{\pm} \rangle = \left(1 + \gamma_{\pm} \alpha_s \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2}\right)\right) S_{\pm} \quad (20)$$

The divergence that appears in this case has been regulated in dimensional regularization ($D = 4 - 2\varepsilon$ dimensions). Requiring

$$A = C_+ \langle Q_+ \rangle + C_- \langle Q_- \rangle \quad (21)$$

we obtain

$$C_{\pm} = 1 + \gamma_{\pm} \alpha_s \ln \frac{M_W^2}{\mu^2} \quad (22)$$

where the divergence has been subtracted in the minimal subtraction scheme. The effective Hamiltonian we have been looking for then reads

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} (C_+(\mu) Q_+ + C_-(\mu) Q_-) \quad (23)$$

with the coefficients C_{\pm} determined in (22) to $\mathcal{O}(\alpha_s \log)$ in perturbation theory. The following points are worth noting:

- The $1/\varepsilon$ (ultraviolet) divergence in the effective theory (20) reflects the $M_W \rightarrow \infty$ limit. This can be seen from the amplitude in the full theory (19), which is finite, but develops a logarithmic singularity in this limit. Consequently, the renormalization in the effective theory is directly linked to the $\ln M_W$ dependence of the decay amplitude.

- We observe that although A and $\langle Q_{\pm} \rangle$ both depend on the long-distance properties of the external states (through p^2), this dependence has dropped out in C_{\pm} . Here we see explicitly how factorization is realized. Technically, to $\mathcal{O}(\alpha_s \log)$, factorization is equivalent to splitting the logarithm of the full amplitude according to

$$\ln \frac{M_W^2}{-p^2} = \ln \frac{M_W^2}{\mu^2} + \ln \frac{\mu^2}{-p^2} \quad (24)$$

Ultimately the logarithms stem from loop momentum integrations and the range of large momenta, between M_W and the factorization scale μ , is indeed separated into the Wilson coefficients.

- To obtain a decay amplitude from \mathcal{H}_{eff} in (23), the matrix elements $\langle f|Q_{\pm}|K \rangle(\mu)$ have to be taken, normalized at a scale μ . An appropriate value for μ is close to the hadronic scale in order not to introduce an unnaturally large scale into the calculation of $\langle Q \rangle$. At the same time μ must also not be too small in order not to render the perturbative calculation of $C(\mu)$ invalid. A typical choice for K decays is $\mu \approx 1 \text{ GeV} \ll M_W$.
- The factorization scale μ is unphysical. It cancels between Wilson coefficient and hadronic matrix element, to a given order in α_s , to yield a scale independent decay amplitude. The mechanism of this cancellation to $\mathcal{O}(\alpha_s)$ is clear from the above example (19) – (22).
- In the construction of \mathcal{H}_{eff} the W -boson is said to be “integrated out”, that is, removed from the effective theory as an explicit degree of freedom. Its effect is still implicitly contained in the Wilson coefficients. The extraction of these coefficients is often called a “matching calculation”, matching the full to the effective theory by “adjusting” the couplings C_{\pm} .
- If we go beyond the leading logarithmic approximation $\mathcal{O}(\alpha_s \log)$ and include the finite corrections of $\mathcal{O}(\alpha_s)$ in (19), (20), an ambiguity arises when renormalizing the divergence in (20) (or, equivalently, in the Wilson coefficients C_{\pm}). This ambiguity consists in what part of the full (non-logarithmic) $\mathcal{O}(\alpha_s)$ term is attributed to the matrix elements, and what part to the Wilson coefficients. In other words, coefficients and matrix elements become *scheme dependent*, that is, dependent on the renormalization scheme, beyond the leading logarithmic approximation. The scheme dependence is unphysical and cancels in the product of coefficients and

matrix elements. Of course, both quantities have to be evaluated in the same scheme to obtain a consistent result. The renormalization scheme is determined in particular by the subtraction constants (minimal or non-minimal subtraction of $1/\varepsilon$ poles), and also by the definition of γ_5 used in $D \neq 4$ dimensions in the context of dimensional regularization.

- Finally, the effective Hamiltonian (23) can be considered as a modern version of the old Fermi theory for weak interactions. It is a systematic low-energy approximation to the standard model for kaon decays and provides the basis for any further analysis.

3.2 Renormalization Group

Let us have a closer look at the Wilson coefficients, which read explicitly

$$C_{\pm} = 1 + \frac{\alpha_s(\mu)}{4\pi} \frac{\gamma_{\pm}^{(0)}}{2} \ln \frac{\mu^2}{M_W^2} \quad \gamma_{\pm}^{(0)} = \begin{cases} 4 \\ -8 \end{cases} \quad (25)$$

where we have now specified the exact form of the $\mathcal{O}(\alpha_s \log)$ correction. Numerically the factor $\alpha_s(\mu)\gamma_{\pm}^{(0)}/(8\pi)$ is about $+7\%$ (-14%), a reasonable size for a perturbative correction (we used $\alpha_s(\mu = 1 \text{ GeV}) = 0.43$). However, this term comes with a large logarithmic factor of $\ln(\mu^2/M_W^2) = -8.8$, for an appropriate scale of $\mu = 1 \text{ GeV}$. The total correction to $C_{\pm} = 1$ in (25) is then -60% (120%)! Obviously, the presence of the large logarithm spoils the validity of a straightforward perturbative expansion, despite the fact that the coupling constant itself is still reasonably small. This situation is quite common in renormalizable quantum field theories. Logarithms appear naturally and can become very large when the problem involves very different scales. The general situation is indicated in the following table, where we display the form of the correction terms in higher orders, denoting $\ell \equiv \ln(\mu/M_W)$

$$\begin{array}{ccc} \text{LL} & \text{NLL} & \\ \alpha_s \ell & \alpha_s & \\ \alpha_s^2 \ell^2 & \alpha_s^2 \ell & \alpha_s^2 \\ \alpha_s^3 \ell^3 & \alpha_s^3 \ell^2 & \alpha_s^3 \ell \quad \alpha_s^3 \\ \downarrow & \downarrow & \\ \mathcal{O}(1) & \mathcal{O}(\alpha_s) & \end{array} \quad (26)$$

In ordinary perturbation theory the expansion is organized according to powers of α_s alone, corresponding to the rows in the above scheme. This approach is invalidated by the large logarithms since $\alpha_s \ell$, in contrast to α_s , is no longer a small parameter, but a quantity of order 1. The problem can be resolved by

resumming the terms $(\alpha_s \ell)^n$ to all orders n . The expansion is then reorganized in terms of columns of the above table. The first column is of $\mathcal{O}(1)$ and yields the leading logarithmic approximation, the second column gives a correction of relative order α_s , and so forth. Technically the reorganization is achieved by solving the renormalization group equation (RGE) for the Wilson coefficients. The RGE is a differential equation describing the change of $C_{\pm}(\mu)$ under a change of scale. To leading order this equation can be read off from (25)

$$\frac{d}{d \ln \mu} C_{\pm}(\mu) = \frac{\alpha_s}{4\pi} \gamma_{\pm}^{(0)} \cdot C_{\pm}(\mu) \quad (27)$$

$(\alpha_s/4\pi)\gamma_{\pm}^{(0)}$ are called the anomalous dimensions of C_{\pm} . To understand the term “dimension”, compare with the following relation for the quantity μ^n , which has (energy) dimension n :

$$\frac{d}{d \ln \mu} \mu^n = n \cdot \mu^n \quad (28)$$

The analogy is obvious. Of course, the $C_{\pm}(\mu)$ are dimensionless numbers in the usual sense; they can depend on the energy scale μ only because there is another scale, M_W , present under the logarithm in (25). Their “dimension” is therefore more precisely called a scaling dimension, measuring the rate of change of C_{\pm} with a changing scale μ . The nontrivial scaling dimension derives from $\mathcal{O}(\alpha_s)$ loop corrections and is thus a genuine quantum effect. Classically the coefficients are scale invariant, $C_{\pm} \equiv 1$. Whenever a symmetry that holds at the classical level is broken by quantum effects, we speak of an “anomaly”. Hence, the $\gamma_{\pm}^{(0)}$ represent the anomalous (scaling) dimensions of the Wilson coefficients.

We can solve (27), using

$$\frac{d\alpha_s}{d \ln \mu} = -2\beta_0 \frac{\alpha_s^2}{4\pi} \quad \beta_0 = \frac{33-2f}{3} \quad C_{\pm}(M_W) = 1 \quad (29)$$

and find

$$C_{\pm}(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{\frac{\gamma_{\pm}^{(0)}}{2\beta_0}} = \left[\frac{1}{1 + \beta_0 \frac{\alpha_s(\mu)}{4\pi} \ln \frac{M_W^2}{\mu^2}} \right]^{\frac{\gamma_{\pm}^{(0)}}{2\beta_0}} \quad (30)$$

This is the solution for the Wilson coefficients C_{\pm} in leading logarithmic approximation, that is to leading order in RG improved perturbation theory. The all-orders resummation of $\alpha_s \log$ terms is apparent in the final expression in (30).

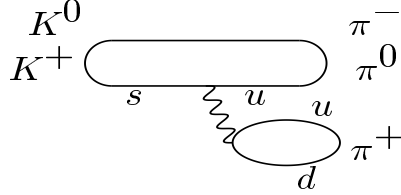


Figure 10: $K^0 \rightarrow \pi^+\pi^-$ and $K^+ \rightarrow \pi^+\pi^0$.

3.3 $\Delta I = 1/2$ Rule

At this point, and before continuing with the construction of the complete $\Delta S = 1$ Hamiltonian, it is interesting to discuss a first application of the results we have derived so far.

Let us consider the weak decays into two pions of a neutral kaon, $K_S \rightarrow \pi^+\pi^-$, and a charged kaon, $K^+ \rightarrow \pi^+\pi^0$, which are sketched in Fig. 10. The two cases look very much the same, except that the spectator quark is a u quark for the charged kaon and a d quark for the neutral one. Naively one would therefore expect very similar decay rates. The experimental facts are, however, strikingly different:

$$\frac{\Gamma(K_S \rightarrow \pi^+\pi^-)}{\Gamma(K^+ \rightarrow \pi^+\pi^0)} \approx 450 \approx (21.2)^2 \quad (31)$$

To get a hint as to where this huge difference in the decay rates may come from we have to analyze the isospin structure of the decays. A kaon state has isospin $I = 1/2$. Taking into account Bose symmetry, one finds that two pions from the decay of a K meson can only be in a state of isospin 0 and 2. More specifically, $|\pi^+\pi^-\rangle$ has both $I = 0$ and $I = 2$ components, while $|\pi^+\pi^0\rangle$ is a pure $I = 2$ state. The change in isospin is then as follows

$$K^+ \rightarrow \pi^+\pi^0 \quad \Delta I = 3/2 \quad (32)$$

$$K^0 \rightarrow \pi^+\pi^- \quad \Delta I = 1/2, 3/2 \quad (33)$$

In particular, $K^+ \rightarrow \pi^+\pi^0$ is a pure $\Delta I = 3/2$ transition. The large ratio in (31) means that $\Delta I = 1/2$ transitions are strongly enhanced. This empirical feature is referred to as the $\Delta I = 1/2$ rule.

We next take a closer look at the isospin properties of the effective Hamiltonian. Using the Fierz identities of the Dirac matrices the operators Q_{\pm} can

be rewritten as

$$\begin{aligned} Q_{\pm} &= (\bar{s}_i u_i)_{V-A} (\bar{u}_j d_j)_{V-A} \pm (\bar{s}_i u_j)_{V-A} (\bar{u}_j d_i)_{V-A} = \\ &= (\bar{s}_i u_i)_{V-A} (\bar{u}_j d_j)_{V-A} \pm (\bar{s}_i d_i)_{V-A} (\bar{u}_j u_j)_{V-A} \end{aligned} \quad (34)$$

where now all quark bilinears appear uniformly as colour singlets. Retaining only the flavour structure, but dropping colour and Dirac labels for ease of notation, the Hamiltonian (23) has the form

$$\begin{aligned} \mathcal{H}_{eff} &\sim \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{6/25} ((\bar{s}u)(\bar{u}d) + (\bar{s}d)(\bar{u}u)) + \\ &+ \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{-12/25} ((\bar{s}u)(\bar{u}d) - (\bar{s}d)(\bar{u}u)) \end{aligned} \quad (35)$$

We can now see that the operator Q_- , in the second line of (35), is a pure $\Delta I = 1/2$ operator: u and d appear in the combination

$$ud - du \hat{=} |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \quad (36)$$

which has isospin 0. The strange quark is also an isospin singlet. The isospin of Q_- is therefore determined by the factor \bar{u} , which has isospin 1/2.

From the Wilson coefficients we have calculated we observe that the contribution from Q_- receives a relative enhancement over Q_+ in (35) by a factor

$$\left[\frac{\alpha_s(\mu)}{\alpha_s(M_W)} \right]^{18/25} \approx \begin{cases} 2.6 & \mu = 1 \text{ GeV} \\ 3.4 & \mu = 0.6 \text{ GeV} \end{cases} \quad (37)$$

Qualitatively, this is precisely what we need: Q_- , which is purely $\Delta I = 1/2$ and can thus only contribute to $K^0 \rightarrow \pi^+\pi^-$, but not to $K^+ \rightarrow \pi^+\pi^0$, is re-inforced by the short-distance QCD dynamics. Quantitatively, however, the RG improved QCD effect falls still short of explaining the amplitude ratio 21.2 in (31) by a sizable factor. We might be tempted to decrease μ , which enhances the effect, but we are not allowed to go much below $\mu = 1 \text{ GeV}$ where perturbation theory would cease to be valid. The remaining enhancement has to come from nonperturbative contributions in the matrix elements. Nevertheless it is interesting to see how already the short-distance QCD corrections provide the first step towards a dynamical explanation of the $\Delta I = 1/2$ rule.

3.4 $\Delta S = 1$ Effective Hamiltonian

In this section we will complete the discussion of the $\Delta S = 1$ effective Hamiltonian. So far we have considered the operators

$$Q_1 = (\bar{s}_i u_j)_{V-A} (\bar{u}_j d_i)_{V-A} \quad (38)$$

$$Q_2 = (\bar{s}_i u_i)_{V-A} (\bar{u}_j d_j)_{V-A} \quad (39)$$

which come from the simple W -exchange graph and the corresponding QCD corrections (Fig. 11). In addition, there is a further type of diagram at $\mathcal{O}(\alpha_s)$,

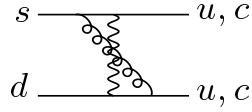


Figure 11: QCD correction to W exchange.

which we have omitted until now: the QCD-penguin diagram shown in Fig. 12. It gives rise to four new operators

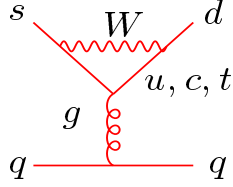


Figure 12: QCD-penguin diagram.

$$Q_3 = (\bar{s}_i d_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A} \quad (40)$$

$$Q_4 = (\bar{s}_i d_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} \quad (41)$$

$$Q_5 = (\bar{s}_i d_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A} \quad (42)$$

$$Q_6 = (\bar{s}_i d_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} \quad (43)$$

Two structures appear when the light-quark current $(\bar{q}q)_V$ from the bottom end of the diagram is split into $V - A$ and $V + A$ parts. In turn, each of those comes in two colour forms in a way similar to Q_1 and Q_2 .

The operators Q_1, \dots, Q_6 mix under renormalization, that is the RGE for their Wilson coefficients is governed by a matrix of anomalous dimensions,

generalizing (27). In this way the RG evolution of $C_{1,2}$ affects the evolution of C_3, \dots, C_6 . On the other hand $C_{1,2}$ remain unchanged in the presence of the penguin operators Q_3, \dots, Q_6 , so that the results for $C_{1,2}$ derived above are still valid.

For some applications (e.g. ε'/ε) higher order electroweak effects need to be taken into account. They arise from γ - or Z -penguin diagrams (Fig. 13) and also from W -box diagrams. Four additional operators arise from this

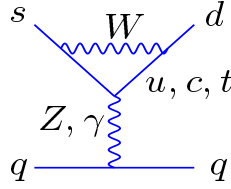


Figure 13: Electroweak penguin.

source. They have a form similar to the QCD penguins, but a different isospin structure, and read (e_q are the quark charges, $e_{u,c} = +2/3$, $e_{d,s,b} = -1/3$)

$$Q_7 = \frac{3}{2}(\bar{s}_i d_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V+A} \quad (44)$$

$$Q_8 = \frac{3}{2}(\bar{s}_i d_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V+A} \quad (45)$$

$$Q_9 = \frac{3}{2}(\bar{s}_i d_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V-A} \quad (46)$$

$$Q_{10} = \frac{3}{2}(\bar{s}_i d_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A} \quad (47)$$

The construction of the effective Hamiltonian follows the principles we have discussed in the previous sections. First the Wilson coefficients $C_i(\mu_W)$, $i = 1, \dots, 10$, are determined at a large scale $\mu_W = \mathcal{O}(M_W, m_t)$ to a given order in perturbation theory. In this step both the W boson and the heavy top quark are integrated out. Since the renormalization scale is chosen to be $\mu_W = \mathcal{O}(M_W, m_t)$, no large logarithms appear and straightforward perturbation theory can be used for the matching calculation. The anomalous dimensions are computed from the divergent parts of the operator matrix elements, which correspond to the UV-renormalization of the Wilson coefficients. Solving the RGE the C_i are evolved from μ_W to a scale $\mu_b = \mathcal{O}(m_b)$ in a theory

with $f = 5$ active flavours $q = u, d, s, c, b$. At this point the b quark (which can appear in loops) is integrated out by calculating the matching conditions from a five-flavour to a four-flavour theory, where only $q = u, d, s, c$ are active. This procedure is repeated by integrating out charm at $\mu_c = \mathcal{O}(m_c)$ and matching onto an $f = 3$ flavour theory. One finally obtains the coefficients $C_i(\mu)$ at a scale $\mu < \mu_c$, describing an effective theory where only $q = u, d, s$ (and gluons of course) are active degrees of freedom. The terms taken into account in the RG improved perturbative evaluation of $C_i(\mu)$ are, schematically:

$$\begin{aligned} \text{LO: } & \left(\alpha_s \ln \frac{M_W}{\mu} \right)^n, \alpha \ln \frac{M_W}{\mu} \left(\alpha_s \ln \frac{M_W}{\mu} \right)^n \\ \text{NLO: } & \alpha_s \left(\alpha_s \ln \frac{M_W}{\mu} \right)^n, \alpha \left(\alpha_s \ln \frac{M_W}{\mu} \right)^n \end{aligned}$$

at leading and next-to-leading order, respectively. Here α is the QED coupling, referring to the electroweak corrections.

The final result for the $\Delta S = 1$ effective Hamiltonian (with 3 active flavours) can be written as

$$\mathcal{H}_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \lambda_u \sum_{i=1}^{10} \left(z_i(\mu) - \frac{\lambda_t}{\lambda_u} y_i(\mu) \right) Q_i + \text{h.c.} \quad (48)$$

where $\lambda_p \equiv V_{ps}^* V_{pd}$. In principle there are three different CKM factors, λ_u , λ_c and λ_t , corresponding to the different flavours of up-type quarks that can participate in the charged-current weak interaction. Using CKM unitarity, one of them can be eliminated. If we eliminate λ_c , we arrive at the CKM structure of (48).

The Hamiltonian in (48) is the basis for computing nonleptonic kaon decays within the standard model, in particular for the analysis of direct CP violation. When new physics is present at some higher energy scale, the effective Hamiltonian can be derived in an analogous way. The matching calculation at the high scale μ_W will give new contributions to the coefficients $C_i(\mu_W)$, the initial conditions for the RG evolution. In general, new operators may also be induced. The Wilson coefficients z_i and y_i are known in the standard model at NLO. A more detailed account of $\mathcal{H}_{eff}^{\Delta S=1}$ and information on the technical aspects of the necessary calculations can be found in¹² and¹³.

3.5 Chiral Perturbation Theory

An additional tool for kaon physics, complementary to the OPE-based effective Hamiltonian formalism, is chiral perturbation theory (χ PT). The present section gives a brief and elementary introduction into this subject. For other, more detailed discussions we refer the reader to^{6,16,17,18}.

Preliminaries

The QCD Lagrangian for three light flavours $q = (u, d, s)^T$ can be written in terms of left-handed and right-handed fields, $q_{L,R} = (1 \mp \gamma_5)q/2$, in the form

$$\mathcal{L}_{QCD} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M}^\dagger q_L \quad (49)$$

where $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$. If \mathcal{M} is put to zero, \mathcal{L}_{QCD} is invariant under a global $SU(3)_L \otimes SU(3)_R$ symmetry

$$q_L \rightarrow L q_L \quad (50)$$

$$q_R \rightarrow R q_R \quad (51)$$

with L and R (independent) $SU(3)$ transformations. The explicit breaking of this *chiral symmetry* through a nonzero \mathcal{M} is a small effect and can be treated as a perturbation. Simultaneously, chiral symmetry is not reflected in the hadronic spectrum, so it must also be spontaneously broken by the dynamics of QCD. For instance the octet of light pseudoscalar mesons

$$\Phi \equiv T^a \pi^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \quad (52)$$

is not accompanied in the spectrum of hadrons by a similar octet of mesons with opposite parity and comparable mass. On the other hand the octet Φ , comprising the lightest existing hadrons, is the natural candidate for the octet of Goldstone bosons expected from the pattern of spontaneous chiral symmetry breaking

$$SU(3)_L \otimes SU(3)_R \rightarrow SU(3) \quad (53)$$

down to group of ordinary flavour $SU(3)$, where $q_{L,R} \rightarrow U q_{L,R}$.

The mesons in Φ are not strictly massless due to the explicit breaking of chiral symmetry caused by \mathcal{M} and are thus often referred to as pseudo-Goldstone bosons. Still they are the lightest hadrons and they are separated by a mass gap from the higher excitations of the light-hadron spectrum. (The masses of the latter remain of order Λ_{QCD} , while the masses of Φ vanish in the limit $\mathcal{M} \rightarrow 0$.)

The idea of χ PT is to write a low-energy effective theory where the only dynamical degrees of freedom are the eight pseudo-Goldstone bosons. This is appropriate for low-energy interactions where the higher states are not kinematically accessible. Their virtual presence will however be contained in the coupling constants of χ PT. The guiding principles for the construction of χ PT

are the chiral symmetry of QCD and an expansion in powers of momenta and quark masses. By constructing the most general Lagrangian for Φ compatible with the symmetries of QCD, the framework is *model independent*. By restricting the accuracy to a given order in the momentum expansion, only a finite number of terms are possible and the framework becomes also *predictive*. A finite number of couplings needs to be fixed from experiment; once this is done, predictions can be made. χ PT is a nonperturbative approach as it does not rely on any expansion in the QCD coupling α_s .

Both χ PT and the quark-level effective Hamiltonian \mathcal{H}_{eff} are low-energy effective theories applicable to kaon decays. What is the difference and how are these two approaches related? The essential, and obvious, difference is that χ PT is formulated directly in terms of hadrons, \mathcal{H}_{eff} in terms of quarks and gluons. The advantage of χ PT is therefore the direct applicability to physical, hadronic amplitudes, without the need to deal with the complicated hadronic matrix elements of quark-level operators. The advantage of \mathcal{H}_{eff} , on the other hand, is the direct link to short-distance physics, which is encoded in the Wilson coefficients. This type of information is important in the context of CP violation or in the search for new physics. In χ PT such information is hidden in the coupling constants, which are not readily calculable and need to be fixed experimentally. From these considerations it is clear that \mathcal{H}_{eff} is more useful for applications where short-distance physics is essential (CP violation, ε'/ε), whereas χ PT is especially suited to deal with long-distance dominated quantities, which are hard to come by otherwise. To relate the two descriptions directly is not an easy task and has so far not been accomplished. A calculation of the couplings of χ PT from the quark picture, establishing a link between \mathcal{H}_{eff} and χ PT, requires one to solve QCD nonperturbatively, which is not possible at present.

$SU(3)$ Transformations

Before describing the explicit construction of χ PT it will be useful to recall a few important properties of $SU(3)$ transformations and to introduce some convenient notation. We define

$$(q^1, q^2, q^3) \equiv (u, d, s) \quad (q_1, q_2, q_3) \equiv (\bar{u}, \bar{d}, \bar{s}) \quad (54)$$

and by U^i_j the components of a generic $SU(3)$ matrix U . By definition, changing upper into lower indices, and vice versa, corresponds to complex conjugation, thus

$$U_i^j \equiv U^{*i}_j = U^{\dagger j}_i \quad (55)$$

The unitarity of U implies

$$U_k^i U_j^k = U^{\dagger i}_k U_j^k = \delta^i_j \quad (56)$$

$$U^i_k U_j^k = U^i_k U^{\dagger k}_j = \delta^i_j \quad (57)$$

Also,

$$\det U = \varepsilon^{ijk} U^1_i U^2_j U^3_k = 1 \quad (58)$$

The fundamental $SU(3)$ triplet q^i and anti-triplet q_i transform, respectively, as

$$q^i \rightarrow U^i_j q^j \quad (59)$$

$$q_i \rightarrow U_i^j q_j \quad (60)$$

It follows from the above that the singlet $q^k q_k$ as well as the Kronecker symbol δ^i_j and the totally antisymmetric tensor ε^{ijk} are invariant under $SU(3)$ transformations.

Higher dimensional representations can also be built. For example, the traceless tensor

$$S^i_j = q^i q_j - \frac{1}{3} \delta^i_j q^k q_k \quad (61)$$

is an irreducible representation of $SU(3)$. Its eight components constitute an $SU(3)$ octet, which transforms as

$$S^i_j \rightarrow U^i_k U_j^l S^k_l \quad (62)$$

We next define the objects

$$r^i = \varepsilon^{ijk} q_j q_k \quad (63)$$

$$r_i = \varepsilon_{ijk} q^j q^k \quad (64)$$

They transform in the same way as the fundamental triplet and anti-triplet in (59) and (60), respectively. We show this for (63). From (60) and using (55) and (57), we have

$$\begin{aligned} r^i &= \varepsilon^{ijk} q_j q_k \rightarrow \\ &\varepsilon^{ijk} U_j^l U_k^m q_l q_m = U_s^i \varepsilon^{njk} U_n^s U_j^l U_k^m q_l q_m = \\ &U_s^i \varepsilon^{njk} U^{\dagger s}_n U^{\dagger l}_j U^{\dagger m}_k q_l q_m = U_s^i \varepsilon^{slm} (\det U^{\dagger}) q_l q_m = U_s^i r^s \end{aligned} \quad (65)$$

which proves our assertion.

Let us consider two simple applications of this formalism.

a) The meson field Φ in (52) corresponds to the quark-level tensor S^i_j and both transform as octets under $SU(3)$. The connection can be seen by writing

out the components of S^i_j and comparing with (52). One recovers the quark flavour composition of the meson states. For example

$$\Phi^1_2 = \pi^+ \hat{=} S^1_2 = q^1 q_2 = u\bar{d} \quad (66)$$

$$\Phi^3_3 = -\frac{2}{\sqrt{6}}\eta \hat{=} S^3_3 = q^3 q_3 - \frac{1}{3}q^k q_k = -\frac{1}{3}(u\bar{u} + d\bar{d} - 2s\bar{s}) \quad (67)$$

The transformation law for Φ^i_j is the same as for S^i_j in (62) or, in matrix notation,

$$\Phi \rightarrow U\Phi U^\dagger \quad (68)$$

b) In section 3.3 we have seen from the discussion of the $\Delta I = 1/2$ rule that the largely dominant part of the weak Hamiltonian is contributed by the pure $\Delta I = 1/2$ operator Q_- . We will now show the important property that Q_- transforms as the component of an octet under $SU(3)_L$. To see this we first note that the operator

$$Q^i_j = r^i r_j - \frac{1}{3}\delta^i_j r^k r_k \quad (69)$$

is an $SU(3)$ octet. This follows because the r^i in (63), (64) transform as the q^i , and S^i_j in (61) is an octet. We next show that the (2,3) component of Q^i_j indeed has the flavour structure of Q_- :

$$\begin{aligned} Q^2_3 &= r^2 r_3 = (q_3 q_1 - q_1 q_3)(q^1 q^2 - q^2 q^1) \\ &\hat{=} (\bar{s}u)(\bar{u}d) - (\bar{s}d)(\bar{u}u) - (\bar{u}u)(\bar{s}d) + (\bar{u}d)(\bar{s}u) \hat{=} 2Q_- \end{aligned} \quad (70)$$

Since the quark fields in Q_- are all left-handed, we see that Q_- transforms as the (2,3) component of an octet Q^i_j under $SU(3)_L$. Trivially, it is also a singlet under $SU(3)_R$. Hence, Q_- transforms as a component of a $(8_L, 1_R)$ under $SU(3)_L \otimes SU(3)_R$.

The transformation law for Q^i_j in matrix notation is, with $L \in SU(3)_L$,

$$Q \rightarrow LQL^\dagger \quad (71)$$

Including the hermitian conjugate we may write

$$2(Q_- + Q_-^\dagger) \hat{=} Q^2_3 + Q^3_2 = \text{tr } \lambda_6 Q \quad (72)$$

where the trace with the Gell-Mann matrix λ_6 is used to project out the proper components (λ_6 is the matrix with entry 1 at positions (2,3) and (3,2), and 0 otherwise).

It is not hard to see that the penguin operators Q_3, \dots, Q_6 also transform as part of an $(8_L, 1_R)$, in the same way as Q_- .

Chiral Lagrangian

We will now construct explicitly the leading terms of the chiral Lagrangian. Since we have to write down the most general form for this Lagrangian to any given order in the momentum expansion, the specific manner in which chiral symmetry is realized does not matter. The most convenient and standard choice is a nonlinear realization where one introduces the unitary matrix

$$\Sigma = \exp\left(\frac{2i}{f}\Phi\right) \quad (73)$$

as the basic meson field. Here f is the generic decay constant for the light pseudoscalars (we have used a normalization in which $f_\pi = 131 \text{ MeV}$).

The field Σ is taken to transform under $SU(3)_L \otimes SU(3)_R$ as

$$\Sigma \rightarrow L \Sigma R^\dagger \quad (74)$$

with $L \in SU(3)_L$ and $R \in SU(3)_R$. In general, (74) implies a complicated, nonlinear transformation law for the field Φ . However, for the special case of an ordinary $SU(3)$ transformation, where $L = R \equiv U$, (74) becomes equivalent to (68). We thus recover the correct transformation for the octet Φ under ordinary $SU(3)$. We can also see that the vacuum state, which corresponds to $\Phi \rightarrow 0$, hence $\Sigma \rightarrow 1$, is invariant under ordinary $SU(3)$ ($1 \rightarrow U 1 U^\dagger = 1$) as it must be. On the other hand, the vacuum is not invariant under the chiral transformation in (74): $1 \rightarrow L 1 R^\dagger \neq 1$. This corresponds to the spontaneous breaking of chiral symmetry. The field (73) with the transformation (74) therefore has the desired properties to describe the pseudo-Goldstone bosons.

The chiral Lagrangian is constructed as a series in powers of momenta, or equivalently numbers of derivatives

$$\mathcal{L}^{QCD} = \mathcal{L}_2^{QCD} + \mathcal{L}_4^{QCD} + \dots \quad (75)$$

$$\mathcal{L}^{\Delta S=1} = \mathcal{L}_2^{\Delta S=1} + \mathcal{L}_4^{\Delta S=1} + \dots \quad (76)$$

\mathcal{L}^{QCD} describes strong interactions, $\mathcal{L}^{\Delta S=1}$ $\Delta S = 1$ weak interactions, and the subscripts on the right-hand side indicate the number of derivatives.

The lowest order strong interaction Lagrangian has the form

$$\mathcal{L}_2^{QCD} = \frac{f^2}{8} \text{tr} [D_\mu \Sigma D^\mu \Sigma^\dagger + 2B_0(\mathcal{M}\Sigma^\dagger + \Sigma\mathcal{M}^\dagger)] \quad (77)$$

We have written D_μ for the derivative, which we later will generalize to a covariant derivative to include electromagnetism. For the moment we may consider D_μ as an ordinary derivative.

The terms in (77) have to be built to respect the symmetries of the QCD Lagrangian in (49). For $\mathcal{M} = 0$, (49) is chirally invariant and we are thus looking for invariants constructed from Σ in (74). Only trivial terms are possible with zero derivatives, for example $\text{tr}(\Sigma\Sigma^\dagger) = \text{const}$. The leading term comes with two derivatives, as anticipated. The only possible form is $\text{tr}(D_\mu\Sigma D^\mu\Sigma^\dagger)$. Here $D_\mu\Sigma D^\mu\Sigma^\dagger$ transforms as $(8_L, 1_R)$

$$D_\mu\Sigma D^\mu\Sigma^\dagger \rightarrow L D_\mu\Sigma D^\mu\Sigma^\dagger L^\dagger \quad (78)$$

and taking the trace gives an invariant. Another possibility would seem to be $\text{tr}(D^2\Sigma\Sigma^\dagger)$, but this term differs from $\text{tr}(D_\mu\Sigma D^\mu\Sigma^\dagger)$ only by a total derivative.

The second term in (77) breaks chiral symmetry. Its form can be found by noting that the symmetry breaking term proportional to \mathcal{M} in (49) would be invariant if \mathcal{M} was interpreted as an auxiliary field transforming as $\mathcal{M} \rightarrow L\mathcal{M}R^\dagger$. To first order in \mathcal{M} and to lowest order in derivatives this leads to the mass term in (77). For $\mathcal{M} \rightarrow L\mathcal{M}R^\dagger$ it would be invariant. For \mathcal{M} fixed to the diagonal mass matrix it breaks chiral symmetry in the appropriate way. We will soon find that this term indeed counts as two powers of momentum. The second order Lagrangian \mathcal{L}_2^{QCD} is then complete. The factor in front of the first term in (77) is fixed by the requirement that the kinetic term for the mesons be normalized in the canonical way. There are no additional parameters for this contribution. The second term in (77) comes with a coupling B_0 . This new parameter is related to the meson masses. To see this more clearly, we can extract the kinetic terms from (77) by expanding to second order in the field Φ . If we keep, for example, only the contributions with neutral kaons K , \bar{K} , we find (up to an irrelevant additive constant)

$$\mathcal{L}_{2,K^0\text{kin}}^{QCD} = \partial^\mu \bar{K} \partial_\mu K - B_0(m_s + m_d) \bar{K} K \quad (79)$$

From (79) and similar relations for the other mesons we obtain expressions for the pseudo-Goldstone boson masses in terms of the quark masses and the parameter $B_0 = \mathcal{O}(\Lambda_{QCD})$

$$\begin{aligned} m_{K^0}^2 &= B_0(m_s + m_d) \\ m_{K^+}^2 &= B_0(m_s + m_u) \\ m_{\pi^+}^2 &= B_0(m_u + m_d) \end{aligned} \quad (80)$$

The meson masses squared are proportional to linear combinations of quark masses. This also clarifies why one factor of \mathcal{M} is equivalent to two powers of momenta in the usual chiral counting (see (77)).

Expanding (77) beyond second order in Φ we obtain terms describing strong interactions among the mesons, such as π - π scattering.

We next need to determine the form of $\mathcal{L}_2^{\Delta S=1}$. As we have seen in sec. 3.3, the dominant contribution to the weak Hamiltonian comes from the component of an $(8_L, 1_R)$ operator as shown in (72). Since we know empirically from the $\Delta I = 1/2$ rule that the enhancement of this piece of the weak Hamiltonian is quite strong, we shall here make the additional approximation to keep only this contribution and drop the rest (related to the operator Q_+). This is a reasonable approximation for many applications. With the results we have derived so far, it is then easy to write down the correct form for $\mathcal{L}_2^{\Delta S=1}$. The structure with two derivatives and the correct transformation properties as an $(8_L, 1_R)$ is given in (78). According to (72) we simply need to take the trace with λ_6 to obtain the right components. Factoring out basic weak interaction parameters we can write

$$\mathcal{L}_2^{\Delta S=1} = \frac{G_F}{\sqrt{2}} |V_{us}^* V_{ud}| g_8 \frac{f^4}{4} \text{tr } \lambda_6 D_\mu \Sigma D^\mu \Sigma^\dagger \quad (81)$$

This Lagrangian introduces one additional parameter, the octet coupling g_8 . Eq. (81) already contains the usual hermitian conjugate part (see (72)). We have neglected the small CP violating effects and factored out the common leading CKM term $V_{us}^* V_{ud} = V_{us} V_{ud}^* = |V_{us}^* V_{ud}|$.

$K_S \rightarrow \pi^+ \pi^-$ from $\mathcal{L}_2^{\Delta S=1}$

In order to make predictions, we first need to fix the constant g_8 . We can use the dominant nonleptonic decay $K_S \rightarrow \pi^+ \pi^-$ for this purpose. Expanding the interaction term in (81) to third order in Φ , and keeping only K , \bar{K} , π^+ and π^- , we find

$$\text{tr } \lambda_6 \partial \Sigma \partial \Sigma^\dagger = -\frac{4i}{f^3} [\pi^+ \partial \bar{K} \partial \pi^- - \partial K \pi^- \partial \pi^+ + (K - \bar{K}) \partial \pi^+ \partial \pi^-] \quad (82)$$

Neglecting CP violation we have $K_1 \equiv K_S$, $K_2 \equiv K_L$, where $CP K_{1,2} = \pm K_{1,2}$, and $(CP K = -\bar{K})$

$$K_{1,2} = \frac{K \mp \bar{K}}{\sqrt{2}} \quad (83)$$

Expressing K , \bar{K} in terms of $K_{1,2}$ we obtain for the square brackets in (82)

$$[\dots] = \frac{1}{\sqrt{2}} \pi^+ \partial \pi^- (\partial K_2 - \partial K_1) - \frac{1}{\sqrt{2}} \pi^- \partial \pi^+ (\partial K_2 + \partial K_1) + \sqrt{2} K_1 \partial \pi^+ \partial \pi^- \quad (84)$$

From (84) the Feynman amplitudes for $K_{1,2} \rightarrow \pi^+\pi^-$ can be read off. Denoting the momentum of the kaon, π^+ , π^- by k , p_1 , p_2 , respectively, we get for K_1

$$[\dots]_{K_1} \rightarrow -\frac{1}{\sqrt{2}} (2p_1 \cdot p_2 + k \cdot (p_1 + p_2)) = -\sqrt{2}(m_K^2 - m_\pi^2) \quad (85)$$

One may check that the corresponding amplitude for $K_2 \rightarrow \pi^+\pi^-$ gives zero, as required by CP symmetry.

From (85) and (81) we obtain the decay amplitude

$$A(K_1 \rightarrow \pi^+\pi^-) = iG_F g_8 |\lambda_u| f_\pi (m_K^2 - m_\pi^2) \quad (86)$$

This gives the branching ratio

$$B(K_S \rightarrow \pi^+\pi^-) = \tau_{K_S} \frac{\sqrt{m_K^2 - 4m_\pi^2}}{16\pi m_K^2} (m_K^2 - m_\pi^2)^2 G_F^2 g_8^2 |\lambda_u|^2 f_\pi^2 \quad (87)$$

Using

$$\tau_{K_S} = 1.3573 \cdot 10^{14} \text{ GeV}^{-1} \quad B(K_S \rightarrow \pi^+\pi^-) = 0.6861 \quad (88)$$

we find

$$g_8 \simeq 5.2 \quad (89)$$

We have thus determined g_8 to lowest order in χ PT. Using this result, we can make predictions. For instance, expanding (81) to fourth order in Φ , we can derive amplitudes for the decays $K \rightarrow 3\pi$. In this manner χ PT relates processes with different numbers of soft pions. Such relations, also known as the soft-pion theorems of current algebra, are nicely summarized in the framework of χ PT in terms of the lowest-order chiral Lagrangians. Other important applications are radiative decays as $K_S \rightarrow \gamma\gamma$, to which we will come back in the following paragraph.

So far we have worked at tree level, which is sufficient at $\mathcal{O}(p^2)$. At the next order, $\mathcal{O}(p^4)$, one has to consider both tree-level contributions of the $\mathcal{O}(p^4)$ terms in the Lagrangians (75), (76), and one-loop diagrams with interactions from the $\mathcal{O}(p^2)$ Lagrangians. The loop diagrams are in general divergent. The divergences are absorbed by renormalizing the couplings at $\mathcal{O}(p^4)$. An example will be described below.

Radiative K Decays

Electromagnetism and the photon field A_μ can be included in χ PT in the usual way. The $U(1)$ gauge transformation for the meson fields is

$$\Sigma' = U \Sigma U^\dagger \Rightarrow \Phi' = U \Phi U^\dagger, \quad U = \exp(-ieQ\Theta) \quad (90)$$

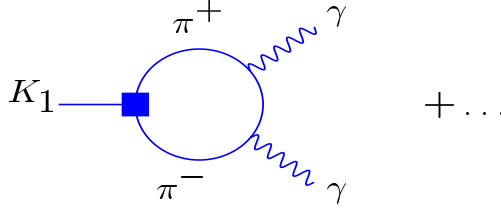


Figure 14: $K_1 \rightarrow \gamma\gamma$ in χ PT.

with $\Theta = \Theta(x)$ an arbitrary real function and the electric charge matrix $Q = \text{diag}(2/3, -1/3, -1/3)$. Writing out (90) for the components of Φ , one finds that each meson transforms with its proper electric charge as the generator. With $A'_\mu = A_\mu - \partial_\mu \Theta$, the covariant derivative that ensures electromagnetic gauge invariance is

$$D_\mu \Sigma = \partial_\mu \Sigma - ie A_\mu [Q, \Sigma] \quad (91)$$

Using this assignment, (77) and (81) include the electromagnetic interactions of the mesons. At higher orders in the chiral Lagrangian also terms with factors of the electromagnetic field strengths $F_{\mu\nu}$ have to be included. In the chiral counting $F_{\mu\nu}$ is equivalent to two powers of momentum.

We finally give a further illustration of the workings of χ PT with two examples of long-distance dominated, radiative kaon decays. We first mention an important theorem for these processes. It says that the amplitudes of nonleptonic radiative kaon decays with *at most one pion* in the final state start only at $\mathcal{O}(p^4)$ in χ PT. This means there are no tree-level contributions at $\mathcal{O}(p^2)$. Such terms are forbidden by gauge invariance. There can only be tree-level amplitudes from $\mathcal{O}(p^4)$, and, at the same order, loop contributions generated from $\mathcal{O}(p^2)$ interactions. Decays that fall under this category are $K \rightarrow \gamma\gamma$, $K \rightarrow \gamma l^+ l^-$, $K \rightarrow \pi\gamma\gamma$ or $K \rightarrow \pi l^+ l^-$.

A particularly interesting example is $K_S \rightarrow \gamma\gamma$. In this case it turns out that there is no direct coupling even at $\mathcal{O}(p^4)$ and hence no counterterm to absorb any divergence from the loop contribution. As a consequence, the one-loop calculation (Fig. 14) is in fact finite. The only parameter involved is g_8 , which we have already determined. The finite loop calculation then gives a unique prediction¹⁹. It yields

$$B(K_S \rightarrow \gamma\gamma) = 2.1 \cdot 10^{-6} \quad (92)$$

This compares well with the experimental result²⁰ $(2.4 \pm 0.9) \cdot 10^{-6}$, which has recently been improved to²¹ $(2.6 \pm 0.4) \cdot 10^{-6}$.

Of course this situation with a finite loop result is somewhat special. A more generic case is $K^+ \rightarrow \pi^+ e^+ e^-$, shown in Fig. 15. Here the loop calcu-

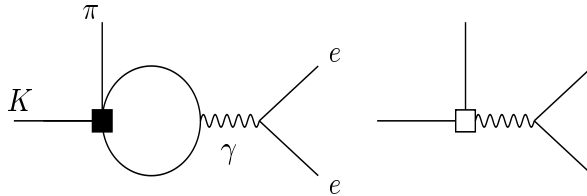


Figure 15: $K^+ \rightarrow \pi^+ e^+ e^-$ in χ PT.

lation is divergent and renormalized by the counterterm at $\mathcal{O}(p^4)$. There is now one additional free parameter, which can be determined from the rate of $K^+ \rightarrow \pi^+ e^+ e^-$. Other observables as the $e^+ e^-$ mass spectrum or the rate and spectrum of $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ can then be predicted. In the same manner one can also analyze the amplitude for $K_S \rightarrow \pi^0 e^+ e^-$, which determines the indirect CP violating contribution in $K_L \rightarrow \pi^0 e^+ e^-$. $K_S \rightarrow \pi^0 e^+ e^-$ is very similar to $K^+ \rightarrow \pi^+ e^+ e^-$, but the required counterterm is different. For this reason the measurement of $K^+ \rightarrow \pi^+ e^+ e^-$ cannot be used to obtain a prediction for $K_S \rightarrow \pi^0 e^+ e^-$. A separate measurement of the latter decay will therefore be needed in the future.

4 The Neutral- K System and CP Violation

4.1 Basic Formalism

Neutral K mesons can mix with their antiparticles through second order weak interactions. They form a two-state system ($K^0 - \bar{K}^0$) that is described by a Hamiltonian matrix \hat{H} of the form

$$\hat{H} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{pmatrix} \quad (93)$$

where CPT invariance has been assumed. The absorptive part Γ_{ij} of \hat{H} accounts for the weak decay of the neutral kaon. In Fig. 16 we show the diagrams that give rise to the off-diagonal elements of \hat{H} . Diagonalizing the Hamiltonian \hat{H} yields the physical eigenstates $K_{L,S}$. They are linear combinations of the strong interaction eigenstates K and \bar{K} and can be written as

$$K_L = \mathcal{N}_\varepsilon [(1 + \varepsilon)K + (1 - \varepsilon)\bar{K}] \equiv pK + q\bar{K} \quad (94)$$

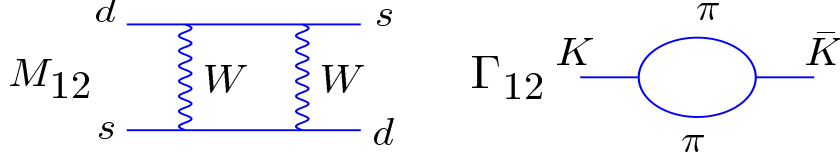


Figure 16: Diagrams contributing to M_{12} and Γ_{12} in the neutral kaon system.

$$K_S = \mathcal{N}_{\bar{\varepsilon}} [(1 + \bar{\varepsilon})K - (1 - \bar{\varepsilon})\bar{K}] \equiv pK - q\bar{K} \quad (95)$$

with the normalization factor $\mathcal{N}_{\bar{\varepsilon}} = 1/\sqrt{2(1 + |\bar{\varepsilon}|^2)}$. Here $\bar{\varepsilon}$ is determined by

$$\frac{1 - \bar{\varepsilon}}{1 + \bar{\varepsilon}} \equiv \frac{q}{p} = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{(\Delta M + \frac{i}{2}\Delta\Gamma)/2} \quad (96)$$

where ΔM and $\Delta\Gamma$ are the differences of the eigenvalues $M_{L,S} - i\Gamma_{L,S}/2$ corresponding to the eigenstates $K_{L,S}$

$$\Delta M \equiv M_L - M_S > 0 \quad \Delta\Gamma \equiv \Gamma_S - \Gamma_L > 0 \quad (97)$$

The labels L and S denote, respectively, the long-lived and the short-lived eigenstate so that $\Delta\Gamma$ is positive by definition. We employ the CP phase convention $CP \cdot K = -\bar{K}$. Using the SM results for M_{12} , Γ_{12} and standard phase conventions for the CKM matrix (see (121)), one finds in the limit of CP conservation ($\eta = 0$) that $\bar{\varepsilon} = 0$. With (94), (95) it follows that K_L is CP odd and K_S is CP even in this limit, which is close to realistic since CP violation is a small effect. As we shall see explicitly later on, the real part of $\bar{\varepsilon}$ is a physical observable, while the imaginary part is not. In particular $(1 - \bar{\varepsilon})/(1 + \bar{\varepsilon})$ is a phase convention dependent, unphysical quantity.

A crucial feature of the kaon system is the very large difference in decay rates between the two eigenstates, the lighter eigenstate decaying much more rapidly than the heavier one ($\Gamma_S = 579\Gamma_L$). The basic reason is the small number of decay channels for the neutral kaons. Decay into the predominant CP even two-pion final states $\pi^+\pi^-$, $\pi^0\pi^0$ is only available for K_S , but not (to first approximation) for the (almost) CP odd state K_L . The latter can decay into three pions, which however is kinematically strongly suppressed, leading to a much longer K_L lifetime.

4.2 Classification of CP Violation

The fundamental weak interaction Lagrangian violates CP invariance via the CKM mechanism, that is through an irreducible complex phase in the quark mixing matrix. This leads to a violation of CP symmetry at the phenomenological level, in particular in the decays of mesons. For instance, processes forbidden by CP symmetry may occur or transitions related to each other by CP conjugation may have a different rate. In order for CP violation to manifest itself in this manner, an interference of some sort between amplitudes is necessary. The interference can arise in a variety of ways. It is therefore useful to introduce a classification of the various possibilities. We shall discuss it in terms of kaons, which are our main concern here, but it is applicable also to D and B mesons in a similar way. According to this classification, which is very common in the literature on CP violation, we may distinguish between:

a) *CP violation in the mixing matrix*. This type of effect is based on CP violation in the two-state mixing Hamiltonian \hat{H} (93) itself and is measured by the observable quantity $\text{Im}(\Gamma_{12}/M_{12})$. It is related to a change in flavour by two units, $\Delta S = 2$.

b) *CP violation in the decay amplitude*. This class of phenomena is characterized by CP violation originating directly in the amplitude for a given decay. It is entirely independent of particle-antiparticle mixing and can therefore occur for charged mesons as well. Here the transitions have $\Delta S = 1$.

c) *CP violation in the interference of mixing and decay*. In this case the interference of two amplitudes takes place between the mixing amplitude and the decay amplitude in decays of neutral mesons. This very important class is sometimes also referred to as *mixing-induced* CP violation, a terminology not to be confused with a).

Complementary to this classification is the widely used notion of *direct* versus *indirect* CP violation. It is motivated historically by the hypothesis of a new superweak interaction which was proposed as early as 1964 by Wolfenstein to account for the CP violation observed in $K_L \rightarrow \pi^+ \pi^-$ decay (see the lectures by Wolfenstein in this volume). This new CP violating interaction would lead to a local four-quark vertex that changes the flavour quantum number (strangeness) by two units. Its only effect would be a CP violating contribution to M_{12} , so that all observed CP violation could be attributed to particle-antiparticle mixing alone. Today, after the advent of the three generation SM, the CKM mechanism of CP violation appears more natural. In principle the superweak scenario represents a logical possibility, leading to a different pattern of observable CP violation effects.

Now, any CP violating effect that can be entirely assigned to CP violation in

M_{12} (as for the superweak case) is termed *indirect CP violation*. Conversely, any effect that can not be described in this way and explicitly requires CP violating phases in the decay amplitude itself is called *direct CP violation*. It follows that class a) represents indirect, class b) direct CP violation. Class c) contains aspects of both. In this latter case the magnitude of CP violation observed in any one decay mode (within the neutral kaon system, say) could by itself be ascribed to mixing, thus corresponding to an indirect effect. On the other hand, a difference in the degree of CP violation between two different modes would reveal a direct effect.

We illustrate these classes by a few important examples. We will also use this opportunity to discuss several aspects of kaon CP violation in more detail.

a) – Lepton Charge Asymmetry

The lepton charge asymmetry in semileptonic K_L decay is an example for CP violation in the mixing matrix. It is probably the most obvious manifestation of CP nonconservation in kaon decays. The observable considered here reads ($l = e$ or μ)

$$\begin{aligned}\Delta &= \frac{\Gamma(K_L \rightarrow \pi^- l^+ \nu) - \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})}{\Gamma(K_L \rightarrow \pi^- l^+ \nu) + \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})} = \frac{|1 + \bar{\varepsilon}|^2 - |1 - \bar{\varepsilon}|^2}{|1 + \bar{\varepsilon}|^2 + |1 - \bar{\varepsilon}|^2} \\ &\approx 2\text{Re } \bar{\varepsilon} \approx \frac{1}{4} \text{Im} \frac{\Gamma_{12}}{M_{12}}\end{aligned}\quad (98)$$

If CP was a good symmetry of nature, K_L would be a CP eigenstate and the two processes compared in (98) were related by a CP transformation. The rate difference Δ should vanish. Experimentally one finds however²⁰

$$\Delta_{exp} = (3.27 \pm 0.12) \cdot 10^{-3} \quad (99)$$

a clear signal of CP violation. The second equality in (98) follows from (94), noting that the positive lepton l^+ can only originate from $K \sim (\bar{s}d)$, l^- only from $\bar{K} \sim (\bar{d}s)$. This is true to leading order in SM weak interactions and holds to sufficient accuracy for our purpose. The charge of the lepton essentially serves to tag the strangeness of the K , thus picking out either only the K or only the \bar{K} component. Any phase in the semileptonic amplitudes is irrelevant and the CP violation effect is purely in the mixing matrix itself. In fact, as indicated in (98), Δ is determined by $\text{Im}(\Gamma_{12}/M_{12})$, the physical measure of CP violation in the mixing matrix.

From (99) we see that $\Delta > 0$. This empirical fact can be used to define positive electric charge in an absolute sense. Positive charge is the charge of

the lepton more copiously produced in semileptonic K_L decay. This definition is unambiguous and would even hold in an antimatter world. Also, using some parity violation experiment, this result implies in addition an absolute definition of left and right. These are quite remarkable facts. They clearly provide part of the motivation to try to learn more about the origin of CP violation.

b) – CP Violation in the Decay Amplitude

Observable CP violation may also occur through interference effects in the decay amplitudes themselves (pure direct CP violation). This case is conceptually perhaps the simplest mechanism for CP violation and the basic features are here particularly transparent. Consider a situation where two different components contribute to the amplitude of a K meson decaying into a final state f

$$A \equiv A(K \rightarrow f) = A_1 e^{i\delta_1} e^{i\phi_1} + A_2 e^{i\delta_2} e^{i\phi_2} \quad (100)$$

Here A_i ($i = 1, 2$) are real amplitudes and δ_i are complex phases from CP conserving interactions. The δ_i are usually strong interaction rescattering phases. Finally the ϕ_i are weak phases, that is phases coming from the CKM matrix in the SM. The corresponding amplitude for the CP conjugated process $\bar{K} \rightarrow \bar{f}$ then reads (the explicit minus signs are due to our convention $CP \cdot K = -\bar{K}$, $(CP \cdot f = \bar{f})$)

$$\bar{A} \equiv A(\bar{K} \rightarrow \bar{f}) = -A_1 e^{i\delta_1} e^{-i\phi_1} - A_2 e^{i\delta_2} e^{-i\phi_2} \quad (101)$$

Since now all quarks are replaced by antiquarks (and vice versa) compared to (100), the weak phases change sign. The CP invariant strong phases remain the same. From (100) and (101) one finds immediately

$$|A|^2 - |\bar{A}|^2 \sim A_1 A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2) \quad (102)$$

The conditions for a nonvanishing difference between the decay rates of $K \rightarrow f$ and the CP conjugate $\bar{K} \rightarrow \bar{f}$, that is direct CP violation, can be read off from (102). There need to be two interfering amplitudes A_1 , A_2 and these amplitudes must simultaneously have both different weak (ϕ_i) and different strong phases (δ_i). Although the strong interaction phases can of course not generate CP violation by themselves, they are still a necessary requirement for the weak phase differences to show up as observable CP asymmetries. It is obvious from (100) and (101) that in the absence of strong phases A and \bar{A} would have the same absolute value despite their different weak phases, since then $A = -\bar{A}^*$.

A specific example is given by the decays $K(\bar{K}) \rightarrow \pi^+\pi^-$ (here $f = \pi^+\pi^- = \bar{f}$). The amplitudes can be written as

$$\begin{aligned} A_{+-} &= \sqrt{\frac{2}{3}}A_0e^{i\delta_0} + \frac{1}{\sqrt{3}}A_2e^{i\delta_2} \\ \bar{A}_{+-} &= -\sqrt{\frac{2}{3}}A_0^*e^{i\delta_0} - \frac{1}{\sqrt{3}}A_2^*e^{i\delta_2} \end{aligned} \quad (103)$$

where $A_{0,2}$ are the transition amplitudes of K to the isospin-0 and isospin-2 components of the $\pi^+\pi^-$ final state, defined by

$$\langle \pi\pi(I=0,2) | \mathcal{H}_W | K \rangle \equiv A_{0,2}e^{i\delta_{0,2}} \quad (104)$$

They still include the weak phases, but the strong phases have been factored out and written explicitly in (103), (104). Taking the modulus squared of the amplitudes we get

$$\begin{aligned} \frac{\Gamma(K \rightarrow \pi^+\pi^-) - \Gamma(\bar{K} \rightarrow \pi^+\pi^-)}{\Gamma(K \rightarrow \pi^+\pi^-) + \Gamma(\bar{K} \rightarrow \pi^+\pi^-)} &= \sqrt{2} \sin(\delta_0 - \delta_2) \frac{\text{Re}A_2}{\text{Re}A_0} \left(\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right) \\ &= 2 \text{Re } \varepsilon' \end{aligned} \quad (105)$$

The quantity so defined is just twice the real part of the famous parameter ε' , the measure of direct CP violation in $K \rightarrow \pi\pi$ decays. The real parts of $A_{0,2}$ can be extracted from experiment. The imaginary parts have to be calculated using the effective Hamiltonian formalism.

We should stress that the quantity in (105) is not the observable actually used to determine ε' experimentally. We have discussed it here because it is of conceptual interest as the simplest manifestation of ε' . The realistic analysis requires a more general consideration of $K_L, K_S \rightarrow \pi\pi$ decays to which we turn in the following paragraph.

c) - Mixing Induced CP Violation in $K \rightarrow \pi\pi$: $\varepsilon, \varepsilon'$

In this section we will illustrate the concept of mixing-induced CP violation with the example of $K \rightarrow \pi\pi$ decays. These are important processes, since CP violation has first been seen in $K_L \rightarrow \pi^+\pi^-$ and as of today our most precise experimental knowledge about this phenomenon still comes from the study of $K \rightarrow \pi\pi$ transitions. There are two distinct final states and in a strong interaction eigenbasis the transitions are $K^0, \bar{K}^0 \rightarrow \pi\pi(I=0), \pi\pi(I=2)$, with definite isospin for $\pi\pi$. Alternatively, using the physical eigenbasis for both initial and final states, one has $K_L, K_S \rightarrow \pi^+\pi^-, \pi^0\pi^0$.

Consider next the amplitude for K_L going into the CP even state $\pi\pi(I=0)$, which can proceed via K ($\sim (1+\bar{\varepsilon})A_0$) or via \bar{K} ($\sim (1-\bar{\varepsilon})A_0^*$). Hence (to first order in small quantities)

$$A(K_L \rightarrow \pi\pi(I=0)) \sim (1+\bar{\varepsilon})A_0e^{i\delta_0} - (1-\bar{\varepsilon})A_0^*e^{i\delta_0} \sim \bar{\varepsilon} + i\frac{\text{Im}A_0}{\text{Re}A_0} = \varepsilon \quad (106)$$

This defines the parameter ε , characterizing mixing-induced CP violation. Note that ε involves a component from mixing ($\bar{\varepsilon}$) as well as from the decay amplitude ($\text{Im}A_0/\text{Re}A_0$). Neither of those is physical separately, but ε is. Note also that the physical quantity $\text{Re}\bar{\varepsilon}$ discussed above satisfies $\text{Re}\bar{\varepsilon} = \text{Re}\varepsilon$. More generally one can form the following two CP violating observables

$$\eta_{+-} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} \quad \eta_{00} = \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)} \quad (107)$$

These amplitude ratios involve the physical initial and final states and are directly measurable in experiment. They are related to ε and ε' through

$$\eta_{+-} = \varepsilon + \varepsilon' \quad \eta_{00} = \varepsilon - 2\varepsilon' \quad (108)$$

The phase of ε is given by $\varepsilon = |\varepsilon|\exp(i\pi/4)$. The relative phase between ε' and ε can be determined theoretically. It is close to zero so that to very good approximation $\varepsilon'/\varepsilon = \text{Re}(\varepsilon'/\varepsilon)$.

Both η_{+-} and η_{00} measure mixing-induced CP violation (interference between mixing and decay). Each of them considered separately could be attributed to CP violation in K - \bar{K} mixing and would therefore represent indirect CP violation. On the other hand, a nonvanishing difference $\eta_{+-} - \eta_{00} = 3\varepsilon' \neq 0$ is a signal of direct CP violation. Experimentally one has²⁰

$$|\varepsilon| = (2.282 \pm 0.019) \cdot 10^{-3} \quad (109)$$

The quantity ε' can be measured as the ratio $\text{Re}(\varepsilon'/\varepsilon) \approx \varepsilon'/\varepsilon$ using the double ratio of rates

$$\left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 \doteq 1 + 6 \text{Re} \frac{\varepsilon'}{\varepsilon} \quad (110)$$

Ten years ago, and until recently, the experimental situation was characterized by the following, somewhat inconclusive results^{22,23}:

$$\text{Re} \frac{\varepsilon'}{\varepsilon} = \begin{cases} (23 \pm 7) \cdot 10^{-4} & \text{CERN NA31} \\ (7.4 \pm 5.9) \cdot 10^{-4} & \text{FNAL E731} \end{cases} \quad (111)$$

In particular the second measurement was well compatible with zero. A new round of experiments, conducted at both CERN and Fermilab, was therefore

anticipated with great interest. The recent results have firmly established direct CP violation^{24,25}:

$$\text{Re} \frac{\varepsilon'}{\varepsilon} = \begin{cases} (28.0 \pm 4.1) \cdot 10^{-4} & \text{FNAL KTeV} \\ (14.0 \pm 4.3) \cdot 10^{-4} & \text{CERN NA48} \end{cases} \quad (112)$$

These results rule out the superweak hypothesis, at least in its most stringent form. The analyses of the experiments are currently still ongoing and should eventually settle the value of ε'/ε to an accuracy of $(1-2) \cdot 10^{-4}$.

4.3 Theory of ε and the Unitarity Triangle

Calculation of ε

In the theoretical expression for ε in (106), the term $\text{Im}A_0/\text{Re}A_0$ is numerically negligible (in standard phase convention). The value for ε is then approximately given by $\bar{\varepsilon}$ from (96), and can be written as

$$\varepsilon = e^{i\pi/4} \frac{\text{Im}M_{12}}{\sqrt{2}\Delta M} \quad (113)$$

M_{12} is related to the first diagram shown in Fig. 16. It is given by

$$M_{12} = \frac{1}{2m_K} \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle \quad (114)$$

Here $\mathcal{H}_{eff}^{\Delta S=2}$ is the effective Hamiltonian for $\Delta S = 2$ transitions, which is derived from the box diagrams for M_{12} in Fig. 16 by performing an operator product expansion. In this case there is only a single operator

$$Q^{\Delta S=2} = (\bar{d}s)_{V-A}(\bar{d}s)_{V-A} \quad (115)$$

in the effective Hamiltonian. One obtains explicitly

$$\begin{aligned} \varepsilon = e^{i\pi/4} \frac{G_F^2 M_W^2 f_K^2}{12\pi^2} \frac{m_K}{\sqrt{2}\Delta M_K} B_K \\ \cdot \text{Im} [\lambda_c^{*2} S_0(x_c) \eta_1 + \lambda_t^{*2} S_0(x_t) \eta_2 + 2\lambda_c^* \lambda_t^* S_0(x_c, x_t) \eta_3] \end{aligned} \quad (116)$$

Here $\lambda_i = V_{is}^* V_{id}$, $f_K = 160 \text{ MeV}$ is the kaon decay constant and, at NLO, the bag parameter B_K is defined by

$$B_K = B_K(\mu) [\alpha_s^{(3)}(\mu)]^{-2/9} \left[1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi} J_3 \right] \quad (117)$$

Table 1: NLO results for η_i with $\Lambda_{\overline{MS}}^{(4)} = (325 \pm 110) \text{ MeV}$, $m_c(m_c) = (1.3 \pm 0.05) \text{ GeV}$, $m_t(m_t) = (170 \pm 15) \text{ GeV}$. The third column shows the uncertainty due to the errors in $\Lambda_{\overline{MS}}$ and quark masses. The fourth column indicates the residual renormalization scale uncertainty at NLO in the product of η_i with the corresponding mass dependent function from eq. (116). These products are scale independent up to the order considered in perturbation theory. The central values of the QCD factors at LO are also given for comparison.

	NLO(central)	$\Lambda_{\overline{MS}}, m_q$	scale dep.	NLO ref.	LO(central)
η_1	1.38	$\pm 35\%$	$\pm 15\%$	²⁶	1.12
η_2	0.574	$\pm 0.6\%$	$\pm 0.4\%$	²⁷	0.61
η_3	0.47	$\pm 3\%$	$\pm 7\%$	²⁸	0.35

$$\langle K^0 | (\bar{d}s)_{V-A} (\bar{s}d)_{V-A} | \bar{K}^0 \rangle \equiv \frac{8}{3} B_K(\mu) f_K^2 m_K^2 \quad (118)$$

The index (3) in eq. (117) refers to the number of flavours in the effective theory and $J_3 = 307/162$ (in the so-called NDR scheme¹²).

The Wilson coefficient multiplying B_K in (116) consists of a charm contribution, a top contribution and a mixed top-charm contribution. It depends on the quark masses, $x_i \equiv m_i^2/M_W^2$, through the functions S_0 . The η_i are the corresponding short-distance QCD correction factors (which depend only slightly on quark masses). Detailed definitions can be found in ¹². Numerical values for η_1 , η_2 and η_3 are summarized in Table 1.

Concerning these results the following remarks should be made.

- ε is dominated by the top contribution ($\sim 70\%$). It is therefore rather satisfying that the related short distance part $\eta_2 S_0(x_t)$ is theoretically extremely well under control, as can be seen in Table 1. Note in particular the very small scale ambiguity at NLO, $\pm 0.4\%$ (for $100 \text{ GeV} \leq \mu_t \leq 300 \text{ GeV}$). This intrinsic theoretical uncertainty is much reduced compared to the leading order result where it would be as large as $\pm 9\%$.
- The η_i factors and the hadronic matrix element are not physical quantities by themselves. When quoting numbers it is therefore essential that mutually consistent definitions are employed. The factors η_i described here are to be used in conjunction with the so-called scale- (and scheme-) invariant bag parameter B_K introduced in (117). The last factor on the right-hand side of (117) enters only at NLO. As a numerical example, if the (scale and scheme dependent) parameter $B_K(\mu)$ is given in the NDR scheme at $\mu = 2 \text{ GeV}$, then (117) becomes $B_K = B_K(\text{NDR}, 2 \text{ GeV}) \cdot 1.31 \cdot 1.05$.

- The quantity B_K has to be calculated by non-perturbative methods. A representative range is

$$B_K = 0.80 \pm 0.15 \quad (119)$$

The status of B_K is reviewed in²⁹.

Determination of the Unitarity Triangle

The source of CP violation in the standard model (SM) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix V entering the charged-current weak interaction Lagrangian

$$\mathcal{L}_{CC} = \frac{g_W}{2\sqrt{2}} V_{ij} \bar{u}_i \gamma^\mu (1 - \gamma_5) d_j W_\mu^+ + h.c. \quad (120)$$

where $(u_1, u_2, u_3) \equiv (u, c, t)$, $(d_1, d_2, d_3) \equiv (d, s, b)$ are the mass eigenstates of the six quark flavours and a summation over $i, j = 1, 2, 3$ is understood.

The unitary CKM matrix has the following explicit form

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (121)$$

where the second expression is a convenient parametrization in terms of λ , A , ϱ and η due to Wolfenstein. It is organized as a series expansion in powers of $\lambda = 0.22$ (the sine of the Cabibbo angle) to exhibit the hierarchy among the transitions between generations. The explicit parametrization shown in (121) is valid through order $\mathcal{O}(\lambda^3)$, an approximation that is sufficient for most practical applications. Higher order terms can be taken into account if necessary¹².

The unitarity structure of the CKM matrix is conventionally displayed in the so-called unitarity triangle, Fig. 17 (left). This triangle is a graphical representation of the unitarity relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ (normalized by $-V_{cd}V_{cb}^*$) in the complex plane of Wolfenstein parameters (ϱ, η) . The angles α , β and γ of the unitarity triangle are phase convention independent and can be determined in CP violation experiments. The area of the unitarity triangle, which is proportional to η , is a measure of CP nonconservation in the standard model.

We briefly summarize the main ingredients of the standard analysis of the unitarity triangle, where ε plays a central role. There are 4 independent parameters λ , A , ϱ and η in the CKM matrix, which are determined from 4 measurements as follows:

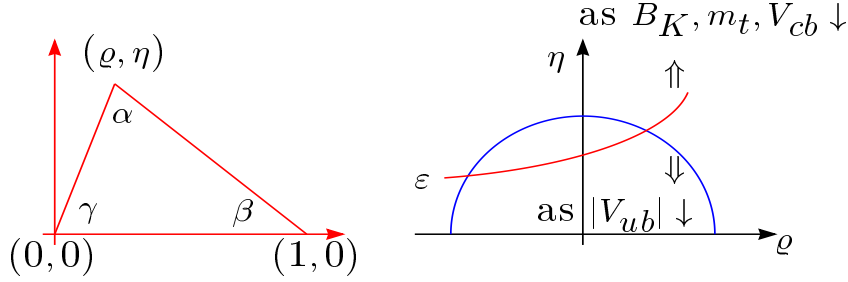


Figure 17: The normalized unitarity triangle in the (ϱ, η) plane (left), and its standard determination (right).

- $\lambda = 0.22$, from $K^+ \rightarrow \pi^0 l^+ \nu$, or from semileptonic hyperon decay ($\Lambda \rightarrow p e \bar{\nu}$, $\Sigma^- \rightarrow n e \bar{\nu}$, $\Xi^- \rightarrow \Lambda e \bar{\nu}$).
- $V_{cb} = A\lambda^2 = 0.040 \pm 0.002$, from $b \rightarrow cl\nu$ transitions.
- $|V_{ub}/V_{cb}| = \lambda\sqrt{\varrho^2 + \eta^2} = 0.09 \pm 0.02$, from $b \rightarrow ul\nu$ transitions.
- $|\varepsilon| \sim \eta \left((1 - \varrho) A^2 S(m_t) + c \right) A^2 B_K$
with $B_K = 0.80 \pm 0.15$, from indirect CP violation in $K \rightarrow \pi\pi$.

Under the final item we have indicated the dependence of ε (from (116)) on the most important parameters, writing the CKM quantities explicitly in Wolfenstein form. Here c denotes a constant that is independent of A , ϱ , η , m_t and B_K .

The standard determination of the unitarity triangle is illustrated in Fig. 17 (right). The relevant input parameters are B_K , m_t , V_{cb} and $|V_{ub}/V_{cb}|$. For fixed B_K , m_t and V_{cb} , the measured $|\varepsilon|$ determines a hyperbola in the ϱ - η plane of Wolfenstein parameters. Intersecting the hyperbola with the circle defined by $|V_{ub}/V_{cb}|$ determines the unitarity triangle (up to a two-fold ambiguity). There is a simple regularity, which is quite useful and easy to remember: As *any* one of the four input parameters becomes *too small* (with the others held fixed), the SM picture becomes inconsistent (see Fig. 17). Using this fact, lower bounds on these parameters can be derived. The large value that has been established for the top-quark mass in fact helps to maintain the consistency of the SM.

In principle, once the unitarity triangle is fixed in this manner, any further, independent measurement of a quantity in the (ϱ, η) plane provides us with

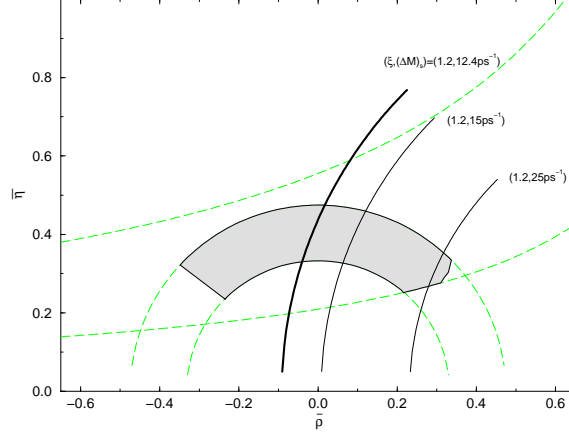


Figure 18: The allowed region (shaded) in the $(\bar{\varrho}, \bar{\eta})$ plane, combining information from ε , $|V_{ub}/V_{cb}|$ and including the constraint from ΔM_d . The independent constraint from the lower limit on $\Delta M_s/\Delta M_d$ excludes the region to the left of the curves labeled with ΔM_s in the plot. $\xi \simeq 1.2$ measures $SU(3)$ breaking in the hadronic matrix elements of B_d - \bar{B}_d versus B_s - \bar{B}_s mixing.

an additional standard model test. In practice, however, the accuracy of such a test is limited by hadronic uncertainties, which enter mainly through B_K and $|V_{ub}/V_{cb}|$. Useful additional restrictions come from B - \bar{B} mixing. Both ΔM_d and $\Delta M_d/\Delta M_s$, where ΔM_q is the mass difference in the B_q - \bar{B}_q system, constrain $\sqrt{(1-\varrho)^2 + \eta^2}$. The ratio $\Delta M_d/\Delta M_s$ is particularly important, because the hadronic uncertainties cancel in the limit of $SU(3)$ symmetry. Currently, while ΔM_d is well measured, only a lower bound $\Delta M_s > 14.3 \text{ ps}^{-1}$ exists at present. However, already this bound is interesting. Together with ΔM_d , it implies a quite clean upper bound on $\sqrt{(1-\varrho)^2 + \eta^2}$. This essentially excludes negative values of ϱ , severely restricting the allowed range of ϱ and η .

The results of a complete analysis of the unitarity triangle are shown in Fig. 18. Here the axes are labeled by $\bar{\varrho} = \varrho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$, instead of ϱ and η . In this way higher terms in the Wolfenstein expansion $\sim \lambda^2$, which can be neglected to first approximation, are consistently taken into account. The plot is taken from³⁰ where further details can be found.

4.4 Calculating ε'/ε

The formula for ε'/ε , which can be derived from the definition in (107), (108), is given by

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}|\varepsilon|} \left(\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right) \quad (122)$$

where $\omega \equiv \text{Re}A_2/\text{Re}A_0$. This may be compared with (105) using

$$\arg(\varepsilon') = \frac{\pi}{2} + \delta_2 - \delta_0 \approx \frac{\pi}{4} \quad (123)$$

The expression (122) for ε'/ε may also be written as

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega}{\sqrt{2}|\varepsilon|\text{Re}A_0} \left(\text{Im}A_0 - \frac{1}{\omega}\text{Im}A_2 \right) \quad (124)$$

$\text{Im}A_{0,2}$ are calculated from the general low energy effective Hamiltonian for $\Delta S = 1$ transitions (48), which we have described in sec. 3.4. One has

$$\text{Im}A_{0,2} = -\text{Im}\lambda_t \frac{G_F}{\sqrt{2}} \sum_{i=3}^{10} y_i(\mu) \langle Q_i \rangle_{0,2} \quad (125)$$

Here y_i are the Wilson coefficients and $\langle Q_i \rangle_{0,2} e^{i\delta_{0,2}} \equiv \langle \pi\pi(I=0,2) | Q_i | K^0 \rangle$, $\lambda_t = V_{ts}^* V_{td}$.

For the purpose of illustration we keep only the numerically dominant contributions and write

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega G_F}{2|\varepsilon|\text{Re}A_0} \text{Im}\lambda_t \left(y_6 \langle Q_6 \rangle_0 - \frac{1}{\omega} y_8 \langle Q_8 \rangle_2 + \dots \right) \quad (126)$$

Q_6 originates from gluonic penguin diagrams and Q_8 from electroweak contributions, as indicated schematically in Fig. 19. The matrix elements of Q_6 and Q_8 can be parametrized by bag parameters B_6 and B_8 as

$$\langle Q_6 \rangle_0 = -4\sqrt{\frac{3}{2}} \left[\frac{m_K}{m_s(\mu) + m_d(\mu)} \right]^2 m_K^2 (f_K - f_\pi) \cdot B_6 \sim \left(\frac{m_K}{m_s} \right)^2 B_6 \quad (127)$$

$$\langle Q_8 \rangle_2 \simeq \sqrt{3} \left[\frac{m_K}{m_s(\mu) + m_d(\mu)} \right]^2 m_K^2 f_\pi \cdot B_8 \sim \left(\frac{m_K}{m_s} \right)^2 B_8 \quad (128)$$

$B_6 = B_8 = 1$ corresponds to the factorization assumption for the matrix elements, which holds in the large N_C limit of QCD.

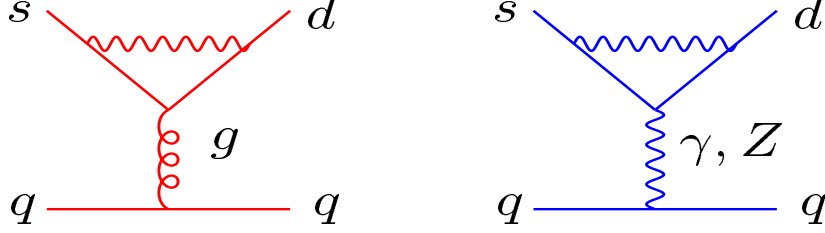


Figure 19: Gluonic and electroweak penguin contributions, which give rise to operators Q_6 and Q_8 , respectively.

The numerical importance of the contributions from Q_6 and Q_8 can be understood as follows. $Q_{6,8}$ are particular because they are of the $(V - A) \otimes (V + A)$ form, which results in a $(S + P) \otimes (S - P)$ structure upon Fierz transformation. Factorizing the matrix element of such operators gives for example

$$\langle \pi^+ \pi^- | (\bar{s}u)_{S+P} (\bar{u}d)_{S-P} | K \rangle \rightarrow -\langle \pi^- | \bar{s}u | K \rangle \cdot \langle \pi^+ | \bar{u} \gamma_5 d | 0 \rangle \quad (129)$$

Taking the derivative (∂^μ) of

$$\langle \pi^+(p) | (\bar{u} \gamma_\mu \gamma_5 d)(x) | 0 \rangle = f_\pi p_\mu e^{ip \cdot x} \quad (130)$$

and using the equations of motion, we find

$$\langle \pi^+ | \bar{u} \gamma_5 d | 0 \rangle = f_\pi \frac{m_\pi^2}{m_u + m_d} = f_\pi \frac{m_K^2}{m_s + m_d} \quad (131)$$

Here the second equality follows from the χ PT relations in (80). A second factor of $m_K^2/(m_s + m_d)$ comes in a similar way from the scalar current matrix element $\langle \pi^- | \bar{s}u | K \rangle$. This explains the quark mass dependence in (127) and (128). Since

$$\frac{m_K^2}{m_s + m_d} = B_0 = \mathcal{O}(\Lambda_{QCD}) \quad (132)$$

we see that the matrix elements are primarily not proportional to $(m_s + m_d)^{-2}$, but to B_0 , which remains finite in the chiral limit $m_s, m_d \rightarrow 0$. However m_K is precisely known and it is customary to trade the χ PT parameter B_0 for the quark masses $m_s + m_d \approx m_s$. Because B_0 is numerically, and somewhat accidentally, quite large (equivalently, the quark masses quite small), the matrix elements of $Q_{6,8}$ are systematically enhanced over those of the ordinary

$(V - A) \otimes (V - A)$ operators. Q_5 and Q_7 have a Dirac structure similar to Q_6 and Q_8 , but a different colour structure, which leads to a $1/N_C$ suppression. In addition their Wilson coefficients are numerically smaller. This implies that Q_6 and Q_8 give the dominant contributions.

$y_6 \langle Q_6 \rangle_0$ and $y_8 \langle Q_8 \rangle_2$ are positive numbers. The value for ε'/ε in (126) is thus characterized by a potential cancellation of two competing contributions. Since the second contribution is an electroweak effect, suppressed by $\sim \alpha/\alpha_s$ compared to the leading gluonic penguin $\sim \langle Q_6 \rangle_0$, it could appear at first sight that it should be altogether negligible for ε'/ε . However, a number of circumstances actually conspire to systematically enhance the electroweak effect so as to render it a sizable contribution:

- Unlike Q_6 , which is a pure $\Delta I = 1/2$ operator, Q_8 can give rise to the $\pi\pi(I = 2)$ final state and thus yield a non-vanishing $\text{Im}A_2$ in the first place.
- The $\mathcal{O}(\alpha/\alpha_s)$ suppression is largely compensated by the factor $1/\omega \approx 22$ in (126), reflecting the $\Delta I = 1/2$ rule.
- $-y_8 \langle Q_8 \rangle_2$ gives a negative contribution to ε'/ε that strongly grows with m_t . For the realistic top mass value it can be substantial.

In order to estimate ε'/ε numerically (see (126)), the hadronic matrix elements have to be determined within a nonperturbative framework (e.g. lattice QCD, $1/N_C$ expansion, chiral quark model), while the coefficients y_i are known from perturbation theory, and $\text{Re}A_0$, ω , G_F , $|\varepsilon|$ are fixed from experiment. Finally, the CKM quantity $\text{Im}\lambda_t \sim \eta$ is obtained from the standard determination of the unitarity triangle described in the previous section.

The Wilson coefficients y_i have been calculated at NLO^{31,32}. The short-distance part is therefore quite well under control. The remaining problem is then the computation of matrix elements, in particular B_6 and B_8 . The cancellation between these contributions enhances the relative sensitivity of ε'/ε to the anyhow uncertain hadronic parameters which makes a precise calculation of ε'/ε impossible at present.

The order of magnitude of ε' can however be understood from (122). The size of $\text{Im}A_i/\text{Re}A_i$ is essentially determined by the small CKM parameters that carry the complex phase and which are related to the top quark in the loop diagrams of Fig. 19. Roughly speaking $\text{Im}A_i/\text{Re}A_i \sim \text{Im}V_{ts}^* V_{td} \sim 10^{-4}$. Empirically we have, from the $\Delta I = 1/2$ rule, $\text{Re}A_2/\text{Re}A_0 \sim 10^{-2}$. This leads to a natural size of ε' of $\sim 10^{-6}$, thus $\varepsilon'/\varepsilon \sim 10^{-3}$.

A complete analysis gives the result^{33,29}

$$1.4 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq 32.7 \cdot 10^{-4} \quad (\text{scanning}) \quad (133)$$

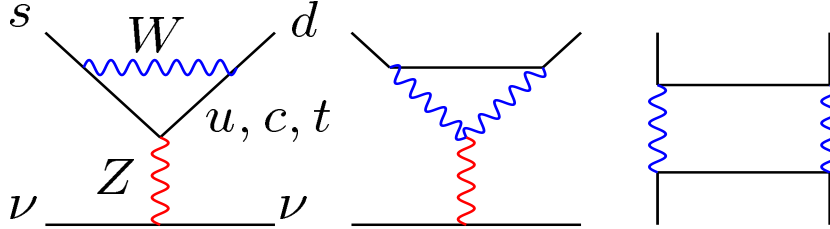


Figure 20: The leading order electroweak diagrams contributing to $K \rightarrow \pi \nu \bar{\nu}$ in the standard model.

$$5.2 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq 16.0 \cdot 10^{-4} \quad (\text{Gaussian}) \quad (134)$$

This is compatible with the experimental results (111), (112) within the rather large uncertainties. The two ranges refer to different treatments of uncertainties in the experimental input: the assumption of Gaussian errors, or flat distributions (scanning). Similar findings are reported by other groups^{34–42}. A detailed review of the theoretical status of ε'/ε , including recent developments (hadronic matrix elements, final state interactions, isospin breaking corrections etc.), along with further references can be found in⁴³.

The recent experimental confirmation that indeed $\varepsilon' \neq 0$ constitutes a qualitatively new feature of CP violation and is as such of great importance. However, due to the large uncertainties in the theoretical calculation, a quantitative use of this result for the extraction of CKM parameters is unfortunately rather limited. For this purpose one has to turn to theoretically cleaner observables. As we will see in the next section, rare K decays in fact offer very promising opportunities in this direction.

5 Rare K Decays

5.1 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

The decays $K \rightarrow \pi \nu \bar{\nu}$ proceed through flavour changing neutral currents. These arise in the standard model only at second (one-loop) order in the electroweak interaction (Z-penguin and W-box diagrams, Fig. 20) and are additionally GIM suppressed. The branching fractions are thus very small, at the level of 10^{-10} , which makes a detection of these modes rather challenging. However, the loop process $K \rightarrow \pi \nu \bar{\nu}$, a genuine quantum effect of standard model flavour dynamics, probes important short distance physics, in particular properties of the top quark (m_t , V_{td} , V_{ts}). It is also very sensitive to poten-

tial new physics effects. At the same time, the $K \rightarrow \pi \nu \bar{\nu}$ modes are reliably calculable, in contrast to most other decay modes of interest. A measurement of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ will therefore be an extremely useful test of flavour physics.

Let us discuss the main properties of these decays, concentrating first on the charged mode. The GIM structure of the amplitude can be written as

$$\sum_{i=u,c,t} \lambda_i F(x_i) = \lambda_c (F(x_c) - F(x_u)) + \lambda_t (F(x_t) - F(x_u)) \quad (135)$$

with $\lambda_i = V_{is}^* V_{id}$ and $x_i = m_i^2/M_W^2$. The first important point is the characteristic *hard* GIM cancellation pattern, which means that the function F depends as a power on the internal mass scale

$$F(x_u) \sim \frac{\Lambda_{QCD}^2}{M_W^2} \sim 10^{-5} \ll F(x_c) \sim \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c} \sim 10^{-3} \ll F(x_t) \sim 1 \quad (136)$$

The up-quark contribution is a long-distance effect, determined by the scale Λ_{QCD} . As an immediate consequence, top and charm contribution with their hard scales m_t , m_c dominate the amplitude, whereas the long-distance part $F(x_u)$ is negligible. Note that the charm contribution, $\lambda_c F(x_c) \sim 10^{-1} \cdot 10^{-3}$, and the top contribution, $\lambda_t F(x_t) \sim 10^{-4} \cdot 1$, have the same order of magnitude when the CKM factors are included.

The origin of the hard GIM mechanism is the fact that the neutrinos only couple to the heavy gauge boson W and Z . It is interesting to contrast the situation with $K^+ \rightarrow \pi^+ e^+ e^-$, where photon exchange can contribute as shown in Fig. 21. For simplicity we consider the case of internal quarks that are light compared to M_W . The W propagator can then be contracted and the loop reduces essentially to a vacuum polarization diagram. Electromagnetic gauge invariance, which is unbroken, requires the $\bar{s}d$ -photon vertex to have the form

$$\Gamma_\nu(q) \sim \bar{s} \gamma^\mu (1 - \gamma_5) d \cdot (q^2 g_{\mu\nu} - q_\mu q_\nu) \ln \frac{M_W}{m_i} \quad (137)$$

where q is the photon momentum and we assumed $q^2 \ll m_i^2 \ll M_W^2$. The structure in (137) ensures current conservation, $q^\nu \Gamma_\nu(q) \equiv 0$. When the vertex is contracted with the electron current $\bar{e} \gamma^\nu e$, the $q_\mu q_\nu$ term vanishes by the equations of motion. The q^2 factor of the remaining term is canceled by the photon propagator, which yields a local $(\bar{s}d)_{V-A}(\bar{e}e)_V$ interaction and a loop function

$$F(x_i) \sim \ln \frac{M_W}{m_i} \quad (138)$$

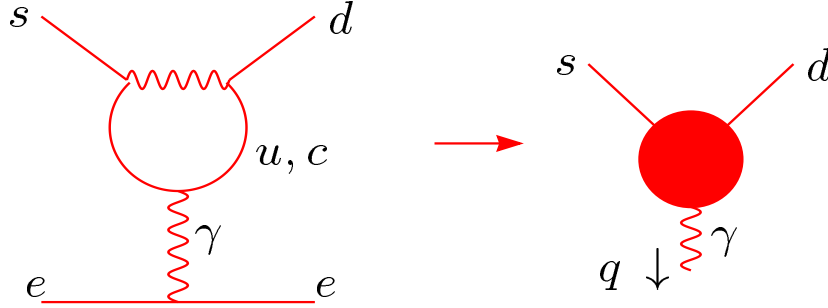


Figure 21: Soft GIM mechanism in the photon penguin.

The logarithmic behaviour of the photon penguin is referred to as the *soft* GIM mechanism. It is in contrast to the hard GIM structure arising from the W and Z contributions where gauge symmetry is spontaneously broken, leading to the power behaviour $\sim (m_i^2/M_W^2) \ln(M_W/m_i)$. The unsuppressed sensitivity to the light quark mass $m_i = m_u$ in (138) is a signal of the long-distance dominance in the $K^+ \rightarrow \pi^+ e^+ e^-$ amplitude. Of course, χ PT is needed for a consistent analysis in this case. The quark-level result in (138), derived in perturbation theory, is strictly speaking not valid, but it is enough to indicate the long-distance sensitivity and the order of magnitude of the contribution.

The short-distance dominance of the $s \rightarrow d \nu \bar{\nu}$ transition next implies that the process is effectively semileptonic, because a single, local operator $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ describes the interaction at low-energy scales. Hence the amplitude has the form

$$A(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \sim G_F \alpha (\lambda_c F_c + \lambda_t F_t) \langle \pi^+ | (\bar{s}d)_V | K^+ \rangle (\bar{\nu}\nu)_{V-A} \quad (139)$$

The coefficient function $\lambda_c F_c + \lambda_t F_t$ is calculable in perturbation theory. The hadronic matrix element can be extracted from $K^+ \rightarrow \pi^0 e^+ \nu$ decay via the isospin relation (13). The $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ amplitude is then completely determined, and with good accuracy.

The neutral mode proceeds through CP violation in the standard model. This is due to the definite CP properties of K^0 , π^0 and the hadronic transition current $(\bar{s}d)_{V-A}$. Using

$$CP|\pi^0\rangle = -|\pi^0\rangle \quad CP|K^0\rangle = -|\bar{K}^0\rangle \quad CP(\bar{s}d)_V(CP)^{-1} = -(\bar{d}s)_V \quad (140)$$

we have

$$\langle \pi^0 | (\bar{s}d)_V | K^0 \rangle = -\langle \pi^0 | (\bar{d}s)_V | \bar{K}^0 \rangle \quad (141)$$

(The axial vector currents $(\bar{s}d)_A$, $(\bar{d}s)_A$ do not contribute to the $K \rightarrow \pi$ transition because of parity.) With $K_L = (K^0 + \bar{K}^0)/\sqrt{2}$ (the $\bar{\varepsilon}$ -contribution is negligible) we then obtain for the matrix element of the hadronic transition current

$$\langle \pi^0 | \lambda_i (\bar{s}d)_V + \lambda_i^* (\bar{d}s)_V | K_L \rangle \sim \text{Im } \lambda_i \quad (142)$$

where λ_i is the appropriate CKM factor. This demonstrates the CP violating character of the leading standard model amplitude for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. For simplicity we have given the argument here assuming standard phase conventions. A manifestly phase convention independent derivation of the same result is discussed in ⁴⁴. The amplitude then has the form

$$A(K_L \rightarrow \pi^0 \nu \bar{\nu}) \sim \text{Im } \lambda_t F_t + \text{Im } \lambda_c F_c \quad (143)$$

where

$$\text{Im } \lambda_t F_t \sim 10^{-4} \cdot 1 \gg \text{Im } \lambda_c F_c \sim 10^{-4} \cdot 10^{-3} \quad (144)$$

The violation of CP symmetry in $K_L \rightarrow \pi^0 \nu \bar{\nu}$ arises through interference between K^0 – \bar{K}^0 mixing and the decay amplitude. This mechanism is an example of mixing-induced CP violation. In the standard model, the mixing-induced CP violation in $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is larger by orders of magnitude than the one in $K_L \rightarrow \pi^+ \pi^-$, for instance. This is because

$$\frac{A(K_L \rightarrow \pi^0 \nu \bar{\nu})}{A(K_S \rightarrow \pi^0 \nu \bar{\nu})} = \mathcal{O}(1) \quad (145)$$

in contrast to the per-mille-size ratios in (107). Any difference in the magnitude of mixing induced CP violation between two K_L decay modes is a signal of direct CP violation. For this reason, the standard model decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is a signal of almost pure direct CP violation, revealing an effect that can not be explained by CP violation in the $K - \bar{K}$ mass matrix alone.

The $K \rightarrow \pi \nu \bar{\nu}$ modes have been studied in great detail over the years to quantify the degree of theoretical precision. Important effects come from short-distance QCD corrections. These were computed at leading order in ⁴⁵. The complete next-to-leading order calculations ^{46,47,48} reduce the theoretical uncertainty in these decays to $\sim 5\%$ for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $\sim 1\%$ for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. This picture is essentially unchanged when further small effects are considered, including isospin breaking in the relation of $K \rightarrow \pi \nu \bar{\nu}$ to $K^+ \rightarrow \pi^0 l^+ \nu$ ⁴⁹, long-distance contributions ^{50,51}, the CP-conserving effect in $K_L \rightarrow$

Table 2: *Compilation of important properties and results for $K \rightarrow \pi \nu \bar{\nu}$.*

	$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$K_L \rightarrow \pi^0 \nu \bar{\nu}$
	CP conserving	CP violating
CKM	V_{td}	$\text{Im} V_{ts}^* V_{td} \sim J_{CP} \sim \eta$
contributions	top and charm	only top
scale dep. (BR)	$\pm 20\%$ (LO) $\rightarrow \pm 5\%$ (NLO)	$\pm 10\%$ (LO) $\rightarrow \pm 1\%$ (NLO)
BR (SM)	$(0.8 \pm 0.3) \cdot 10^{-10}$	$(2.8 \pm 1.1) \cdot 10^{-11}$
exp.	$(1.5^{+3.4}_{-1.2}) \cdot 10^{-10}$ BNL 787 ⁵⁵	$< 5.9 \cdot 10^{-7}$ KTeV ⁵⁶

$\pi^0 \nu \bar{\nu}$ in the standard model^{50,52}, two-loop electroweak corrections for large m_t ⁵³ and subleading-power corrections in the OPE in the charm sector⁵⁴.

While already $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ can be reliably calculated, the situation is even better for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Since only the imaginary part of the amplitude contributes, the charm sector, in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ the dominant source of uncertainty, is completely negligible for $K_L \rightarrow \pi^0 \nu \bar{\nu}$ (0.1% effect on the branching ratio). Long distance contributions ($\lesssim 0.1\%$) and also the indirect CP violation effect ($\lesssim 1\%$) are likewise negligible. The total theoretical uncertainties, from perturbation theory in the top sector and in the isospin breaking corrections, are safely below 3% for $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$. This makes this decay mode truly unique and very promising for phenomenological applications.

In Table 2 we have summarized some of the main features of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Note that the ranges given as the standard model predictions in Table 2 arise from our, at present, limited knowledge of standard model parameters (CKM), and not from intrinsic uncertainties in calculating the branching ratios.

With a measurement of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ available very interesting phenomenological studies could be performed. For instance, $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ together determine the unitarity triangle (Wolfenstein parameters ϱ and η) completely (Fig. 22). The expected accuracy with $\pm 10\%$ branching ratio measurements is comparable to the one that can be achieved by CP violation studies at B factories before the LHC era⁴⁴. The quantity $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ by itself offers probably the best precision in determining $\text{Im} V_{ts}^* V_{td}$ or, equivalently, the Jarlskog parameter

$$J_{CP} = \text{Im}(V_{ts}^* V_{td} V_{us} V_{ud}^*) = \lambda \left(1 - \frac{\lambda^2}{2} \right) \text{Im} \lambda_t \quad (146)$$

The prospects here are even better than for B physics at the LHC. As an

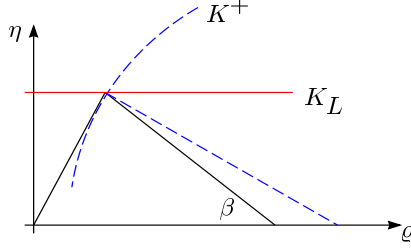


Figure 22: Unitarity triangle from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$.

example, let us assume the following results will be available from B physics experiments

$$\sin 2\alpha = 0.40 \pm 0.04 \quad \sin 2\beta = 0.70 \pm 0.02 \quad V_{cb} = 0.040 \pm 0.002 \quad (147)$$

The small errors quoted for $\sin 2\alpha$ and $\sin 2\beta$ from CP violation in B decays require precision measurements at the LHC. In the case of $\sin 2\alpha$ we have to assume in addition that the theoretical problem of ‘penguin-contamination’ can be resolved. These results would then imply $\text{Im}\lambda_t = (1.37 \pm 0.14) \cdot 10^{-4}$. On the other hand, a $\pm 10\%$ measurement $B(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.3) \cdot 10^{-11}$ together with $m_t(m_t) = (170 \pm 3) \text{ GeV}$ would give $\text{Im}\lambda_t = (1.37 \pm 0.07) \cdot 10^{-4}$. If we are optimistic and take $B(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.15) \cdot 10^{-11}$, $m_t(m_t) = (170 \pm 1) \text{ GeV}$, we get $\text{Im}\lambda_t = (1.37 \pm 0.04) \cdot 10^{-4}$, a remarkable accuracy. The prospects for precision tests of the standard model flavour sector will be correspondingly good.

The charged mode $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is still being studied by Brookhaven experiment E787, which will be followed by a successor experiment, E949⁵⁷. Recently, a new experiment, CKM⁵⁸, has been proposed to measure $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ at the Fermilab Main Injector, studying K decays in flight. Plans to investigate this process also exist at KEK for the Japan Hadron Facility (JHF)⁵⁹.

The neutral mode, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, is currently pursued by KTeV. For $K_L \rightarrow \pi^0 \nu \bar{\nu}$ a model independent upper bound can be inferred from the experimental result on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, which at present is stronger than the direct experimental limit⁶⁰. It is given by $B(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 4.4 B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) < 2 \cdot 10^{-9}$. At least this sensitivity will have to be achieved before new physics is constrained with $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$. Concerning the future of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ experiments, a proposal exists at Brookhaven (BNL E926) to measure this decay at the AGS with a sensitivity of $\mathcal{O}(10^{-12})$ ⁶¹. There are furthermore plans to pursue this mode with comparable sensitivity at Fermilab⁶² and KEK⁶³. Prospects for

$K_L \rightarrow \pi^0 \nu \bar{\nu}$ at a ϕ -factory were discussed in ⁶⁴.

5.2 $K_L \rightarrow \pi^0 e^+ e^-$

The electric charge of the leptons and the resulting interaction with photons make the decay $K_L \rightarrow \pi^0 e^+ e^-$ more complicated than $K_L \rightarrow \pi^0 \nu \bar{\nu}$. The short-distance dominated part of the $K_L \rightarrow \pi^0 e^+ e^-$ amplitude can be analyzed using OPE and the renormalization group in analogy to the case of the $\Delta S = 1$ effective Hamiltonian. This approach is reviewed in ¹². Here we would like to give a qualitative discussion, which highlights the characteristic points and also summarizes the main differences between $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$.

The basic diagrams for $K_L \rightarrow \pi^0 e^+ e^-$ are similar to those for $K_L \rightarrow \pi^0 \nu \bar{\nu}$, except that also a photon can be exchanged instead of the Z boson in the penguin diagrams (see Fig. 20). Taking the GIM mechanism into account, the structure of the effective Hamiltonian reads

$$\mathcal{H}_{eff} \sim \lambda_t (F_t - F_c) + \lambda_u (F_u - F_c) \quad (148)$$

Recalling that we can write $K_L \approx K_2 + \varepsilon K_1$, where K_2 (K_1) is the CP odd (CP even) neutral kaon state, the decay amplitude takes the form

$$A(K_L \rightarrow \pi^0 f \bar{f}) \sim \text{Im} \lambda_t (F_t - F_c) + \varepsilon [\text{Re} \lambda_t (F_t - F_c) + \text{Re} \lambda_u (F_u - F_c)]$$

$$10^{-4} \cdot 1 \quad + 10^{-3} \left[10^{-4} \cdot 1 \quad + 10^{-1} \cdot \begin{cases} 10^{-3} & f = \nu \\ 1 & f = e \end{cases} \right]$$

Here we have kept the expression general to allow for the cases $f = e$ and ν . We have assumed the structure of the short-distance Hamiltonian for this exercise, although this is not strictly correct for the parts that are sensitive to long-distance physics. However it is sufficient for the qualitative argument we would like to make. We recall a few points from the discussion in the section on $K \rightarrow \pi \nu \bar{\nu}$.

First, as we have seen, the K_2 component yields an amplitude proportional to the imaginary parts of the CKM elements. Correspondingly, the K_1 amplitude (opposite CP), which is multiplied by ε , gives the real parts. Second, we need to consider that the GIM mechanism is hard for $f = \nu$ and soft for $f = e$. This implies $F_t \sim 1$, $F_c \sim 10^{-3}$, $F_u \sim 10^{-5}$ for $f = \nu$, and $F_{u,c,t} \sim 1$ for $f = e$. Third, we determine the hierarchy of the CKM elements. Putting this information together, we find the order-of-magnitude estimates shown above for the various terms.

For $f = \nu$ we recover what we already know from the previous section: The amplitude is purely from direct CP violation (the ε -component is negligible)

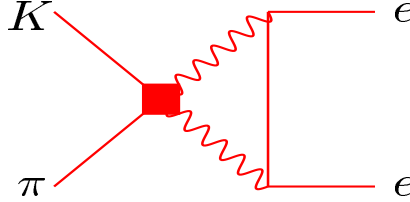


Figure 23: CP conserving two-photon contribution to $K_L \rightarrow \pi^0 e^+ e^-$.

and it is dominated by short-distance physics (only F_t , F_c contribute). For $f = e$ we can read off: Both direct and indirect CP violation contribute at the same order ($\sim 10^{-4}$) and the latter component is determined by long-distance dynamics (F_u).

In addition to what we have discussed so far, a long distance dominated, CP conserving amplitude with two-photon intermediate state can contribute. Although it is of higher order in the electromagnetic coupling, it can potentially compete with the other contributions because those are suppressed by CP violation. Treating $K_L \rightarrow \pi^0 e^+ e^-$ theoretically one is thus faced with the need to disentangle three different contributions of roughly the same order of magnitude.

- Direct CP violation: This amplitude is short-distance in character, theoretically clean and has been analyzed at next-to-leading order in QCD⁶⁵. Taken by itself this mechanism leads to a $K_L \rightarrow \pi^0 e^+ e^-$ branching ratio of $(4.5 \pm 2.6) \cdot 10^{-12}$ within the standard model.
- Indirect CP violation: This part is given by $\sim \varepsilon \cdot A(K_S \rightarrow \pi^0 e^+ e^-)$. The K_S amplitude is dominated by long distance physics and has been investigated in chiral perturbation theory^{66,67,68,69}. Due to unknown counterterms that enter this analysis a reliable prediction is not possible at present. The situation would improve with a measurement of $B(K_S \rightarrow \pi^0 e^+ e^-)$, which could become possible at the CERN experiment NA48 or with KLOE at DAΦNE, the Frascati Φ-factory. Present day estimates for $B(K_L \rightarrow \pi^0 e^+ e^-)$ due to indirect CP violation alone allow values of $10^{-12} - 10^{-10}$.
- The CP conserving two-photon contribution is again long-distance dominated. It has been analyzed by various authors^{68,70,71}. The estimates are typically a few 10^{-12} . Improvements in this sector might be possible

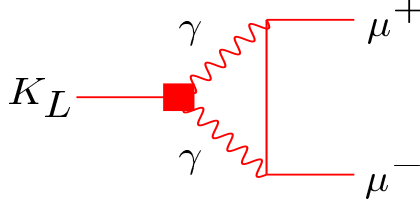


Figure 24: The dominant contribution to $K_L \rightarrow \mu^+ \mu^-$.

by further studying the related decay $K_L \rightarrow \pi^0 \gamma \gamma$ whose branching ratio has already been measured to be $(1.7 \pm 0.1) \cdot 10^{-6}$.

Originally it had been hoped that the direct CP violating contribution would be dominant. It is possible that the CP conserving part is not too important. However, this is unlikely to be true for the amplitude from indirect CP violation. Experimental input on $K_S \rightarrow \pi^0 e^+ e^-$ will be indispensable to solve this problem. We also mention that the CP violating contributions interfere with a relative phase, which is known up to a sign ambiguity. The CP conserving part simply adds incoherently to the decay rate.

Besides the theoretical issues, $K_L \rightarrow \pi^0 e^+ e^-$ is also challenging from an experimental point of view. The expected branching ratio is even smaller than for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Furthermore a serious irreducible physics background from the radiative mode $K_L \rightarrow e^+ e^- \gamma \gamma$ has been identified, which poses additional difficulties¹. A background subtraction seems necessary, which should be possible with enough events. Additional information could in principle also be gained by studying the electron energy asymmetry^{68,71} or the time evolution^{68,72,73}.

5.3 $K_L \rightarrow \mu^+ \mu^-$

$K_L \rightarrow \mu^+ \mu^-$ receives a short distance contribution from Z-penguin and W-box graphs similar to $K \rightarrow \pi \nu \bar{\nu}$. This part of the amplitude is sensitive to the Wolfenstein parameter ϱ . In addition $K_L \rightarrow \mu^+ \mu^-$ proceeds through a long distance contribution with the two-photon intermediate state, which actually dominates the decay completely (Fig. 24). The long distance amplitude consists of a dispersive (A_{dis}) and an absorptive contribution (A_{abs}). The branching fraction can thus be written

$$B(K_L \rightarrow \mu^+ \mu^-) = |A_{SD} + A_{dis}|^2 + |A_{abs}|^2 \quad (149)$$

Using $B(K_L \rightarrow \gamma\gamma)$ it is possible to extract¹ $|A_{abs}|^2 = (7.1 \pm 0.2) \cdot 10^{-9}$. A_{dis} on the other hand cannot be calculated accurately at present^{74,75,76,77}. This is rather unfortunate, in particular since $B(K_L \rightarrow \mu^+\mu^-)$ has already been measured with very good precision

$$B(K_L \rightarrow \mu^+\mu^-) = (7.18 \pm 0.17) \cdot 10^{-9} \text{ BNL 871 }^{78} \quad (150)$$

Interestingly, the absorptive contribution essentially saturates the total rate. For comparison we note that³⁰ $B(K_L \rightarrow \mu^+\mu^-)_{SD} = |A_{SD}|^2 = (0.9 \pm 0.4) \cdot 10^{-9}$ is the expected branching ratio in the standard model based on the short-distance contribution alone. Because A_{dis} is largely unknown, $K_L \rightarrow \mu^+\mu^-$ is at present not a very useful constraint on CKM parameters.

6 Flavour Physics with Charm

6.1 Rare D Decays

Weak decays of charmed particles, D mesons in particular, can also be used to probe the physics of flavour. Due to the characteristic pattern of standard model weak interactions, rare processes with D mesons are markedly different from their kaon counterparts. First of all, the charm quark mass $m_c \approx 1.4$ GeV is considerably bigger than the strange quark mass, and also the QCD scale Λ_{QCD} . Sometimes methods from the theory of heavy quarks can be employed for charmed particles, although they are less reliable than in the case of the much heavier B mesons (see the lectures by Falk in this volume for a general discussion in the context of B physics). This situation makes a theoretical treatment of D decays more difficult than for the heavy B mesons on one hand, and for the light kaons on the other.

More important, however, is the fact that charm is a quark with weak isospin $T_3 = +1/2$, in contrast to s and b . For this reason the virtual particles appearing in FCNC loop diagrams are the down-type quarks d , s , b , rather than u , c , t familiar from K and B physics. Examples of rare D decays are

$$D \rightarrow \rho\gamma, \quad D \rightarrow \pi l^+ l^-, \quad D \rightarrow \mu^+ \mu^-, \quad D \rightarrow \gamma\gamma, \quad D \rightarrow \pi\nu\bar{\nu} \quad (151)$$

They proceed through $c \rightarrow u$ FCNC transitions and their $\Delta C = 1$ amplitudes have the generic form

$$\begin{aligned} & V_{cd}^* V_{ud} F_d + V_{cs}^* V_{us} F_s + V_{cb}^* V_{ub} F_b \\ &= V_{cs}^* V_{us} (F_s - F_d) + V_{cb}^* V_{ub} (F_b - F_d) \end{aligned} \quad (152)$$

where F_i is the amplitude with internal quark $i = d, s, b$. A potentially large contribution could come from F_b , which can have a quadratic mass dependence

$\sim m_b^2/M_W^2$. This exceeds the contribution from light flavours in the loop, but it is still much smaller than virtual top effects. In addition, the b -quark contribution is very strongly CKM suppressed since $V_{cb}^*V_{ub} \sim \lambda^5$. On the other hand, $V_{cs}^*V_{us} \sim \lambda$ is much larger, but this term is multiplied by $F_s - F_d$. The latter is non-zero only through the effects of $SU(3)$ breaking and therefore receives a strong GIM suppression. Moreover, the light-quark loops F_s, F_d are sensitive to nonperturbative QCD dynamics. Additional long-distance mechanisms, different from F_s, F_d , can also become important.

An example is $D^0 \rightarrow \mu^+\mu^-$. Here the amplitude in (152) yields a tiny branching fraction of $\sim 10^{-19}$. Alternatively the decay can proceed, for instance, via the long-distance mechanism $D^0 \rightarrow K^0 \rightarrow \mu^+\mu^-$, with an off-shell kaon. Estimates of this and similar sources give together $B(D^0 \rightarrow \mu^+\mu^-) \sim 10^{-15}$. This still leaves a window for the discovery of potential new physics effects below the current experimental limit of $B(D^0 \rightarrow \mu^+\mu^-)_{exp} < 4 \cdot 10^{-6}$.

6.2 D^0 - \bar{D}^0 Mixing

A further interesting probe of the flavour sector is D^0 - \bar{D}^0 mixing, a process with $\Delta C = 2$. Recent measurements from the CLEO and FOCUS collaborations have stimulated the interest in this observable. A detailed discussion of these results and a list of references can be found in ⁷⁹.

D^0 - \bar{D}^0 mixing can be searched for in the time-dependent analysis of hadronic two-body modes. We may distinguish three types of decays,

$$\text{Cabibbo favoured (CF): } D^0 \rightarrow K^-\pi^+ \quad (153)$$

$$\text{singly Cabibbo suppressed (SCS): } D^0 \rightarrow K^+K^- \quad (154)$$

$$\text{doubly Cabibbo suppressed (DCS): } D^0 \rightarrow K^+\pi^- \quad (155)$$

In powers of the Wolfenstein parameter λ , the CF amplitude $c \rightarrow s\bar{u}d$ is of order λ^0 , the SCS amplitude $c \rightarrow s\bar{u}s$ of order λ , and the DCS amplitude $c \rightarrow d\bar{u}s$ of order λ^2 , which establishes a clear hierarchy in the decay rates. The classification applies of course also to the charge-conjugated modes $\bar{D}^0 \rightarrow K^+\pi^-$ (CF), $\bar{D}^0 \rightarrow K^+K^-$ (SCS) and $\bar{D}^0 \rightarrow K^-\pi^+$ (DCS).

The framework for describing D^0 - \bar{D}^0 mixing is analogous to the case of K^0 - \bar{K}^0 mixing discussed in sec. 4.1. The mass eigenstates are

$$D_{1,2} = pD \pm q\bar{D} \quad CP \cdot D = -\bar{D} \quad (156)$$

We introduce the convenient definitions (Γ is the average total decay rate)

$$x = \frac{M_2 - M_1}{\Gamma} \quad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma} \quad \xi = \frac{p}{q} \frac{\langle K^+\pi^- | D \rangle}{\langle K^+\pi^- | \bar{D} \rangle} \quad (157)$$

x, y, ξ are small quantities of order λ^2 . The precise calculation of x and y is difficult for the reasons mentioned above (GIM suppression, long-distance sensitivity). Analyses based on hadronic estimates (K, π intermediate states) or the heavy-quark expansion (quark-level calculation) lead to a standard model expectation of

$$x, y \lesssim 10^{-4} \quad (158)$$

We then consider the time dependent decay $\Gamma(D^0(t) \rightarrow K^+\pi^-)$. The state $D^0(t)$ is obtained by solving the Schrödinger equation for the D^0 - \bar{D}^0 two-state system with the initial condition $D^0(t=0) = D^0$. The solution reads

$$\begin{aligned} \Gamma(D^0(t) \rightarrow K^+\pi^-) = e^{-\Gamma t} & \left| \frac{q}{p} \right|^2 \Gamma(\bar{D}^0 \rightarrow K^+\pi^-) \\ & \cdot \left[|\xi|^2 + (y \operatorname{Re}\xi + x \operatorname{Im}\xi)\Gamma t + \frac{1}{4}(x^2 + y^2)(\Gamma t)^2 \right] \end{aligned} \quad (159)$$

To obtain this result we have used the following approximations. We expand in the small quantities ξ, x, y , assumed to be of the same order (of order x , say), and drop terms of $\mathcal{O}(x^4)$ and higher. We also require Γt to be at most of $\mathcal{O}(1)$. This is the range that is relevant experimentally, since the D mesons will have decayed once Γt becomes too large.

The form of (159) is not hard to interpret. If there is no mixing, $x = y = 0$, only the first term inside the square brackets survives. It simply describes the exponentially decaying rate of the DCS process $D^0 \rightarrow K^+\pi^-$. Even if $x, y \neq 0$, at time $t = 0$ the DCS decay is the only effect. On the other hand, if the small DCS decay was absent, $\xi = 0$, only the third term contributes. Then the $K^+\pi^-$ final state can arise only through mixing $D^0 \rightarrow \bar{D}^0 \rightarrow K^+\pi^-$. The rate vanishes at $t = 0$ because $D^0(t=0) = D^0$ and there was no time yet for mixing. The $(x\Gamma t)^2$ behaviour represents the onset of an oscillating trigonometric function of which it is the remnant in our approximation. Finally, the linear term $\sim \Gamma t$ is from the interference of DCS decay and mixing.

Without the DCS mode, the observation of $K^+\pi^-$ from an initial D^0 would be an unmistakable sign of mixing. The real situation is more complicated. To demonstrate the presence of the mixing term, the three contributions in (159) have to be disentangled, which is possible in principle due to their different time dependence. So far only the DCS component ($|\xi|^2$ term) has been unambiguously identified. At present the signal for mixing is still compatible with zero. It will be interesting to follow the future experimental results on this issue. The detection of a substantial value of x would indeed be exciting evidence for new physics.

7 Summary and Outlook

Kaon decays have played a key role in the development of the standard model. Today kaon physics is a mature field. A whole array of modern field theoretical techniques is at our disposal to help us extract the underlying mechanisms. Among these tools are perturbative quantum field theory, including the perturbative treatment of QCD at short distances, the operator product expansion, the renormalization group and chiral perturbation theory. While an impressive number of crucial insights has been already obtained in the past, excellent opportunities continue to exist for present and future studies:

- Chiral perturbation theory constitutes a complementary handle on the elusive nonperturbative dynamics of QCD at long distances. This can be helpful to control long-distance contributions that contaminate the short-distance physics that is of primary interest. However, chiral perturbation theory, as a model-independent framework for low-energy QCD, is also of considerable interest in its own right. Typical processes that are studied in this context are $K^+ \rightarrow \pi^+ l^+ l^-$, $K_L \rightarrow \pi^0 \gamma \gamma$, or $K_S \rightarrow \gamma \gamma$.
- CP violation in $K \rightarrow \pi \pi$ is still an important area of current interest. Indirect CP violation measured by ε is well determined experimentally and provides us with a valuable CKM constraint. Direct CP violation, now established, but still under further experimental investigation, gives an important qualitative test of the standard model.
- Standard model precision tests will be possible with the “golden” decay modes $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$.
- Other opportunities of interest include decays as $K_L \rightarrow \pi^0 e^+ e^-$, μ -polarization in $K^+ \rightarrow \pi^0 \mu^+ \nu$, among many other rare processes.
- Very direct and clean probes for new physics are decays that are forbidden in the standard model. Lepton flavour violating modes as $K_L \rightarrow e \mu$, $K \rightarrow \pi \mu e$ are important examples.

In parallel to kaon physics many other observables, as provided from decays of hadrons with beauty and charm, will be necessary to get a reliable and complete picture of the physics of flavour and its possible origins.

In these lectures, we have discussed selected examples from the phenomenology of mesons with strangeness and charm. We have also emphasized the theoretical framework for an analysis of these processes, which is crucial to interpret the experimental data and to extract the underlying physics. The coming years promise to be very fruitful for the study of flavour physics and important discoveries are possible in the near future.

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